SOME STUDIES OF DYNAMICAL SYMMETRY BREAKING IN GAUGE MODELS

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INDIAN INSTITUTE OF TECHNOLOGY, KANPUR
SEPTEMBER, 1984

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A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of

DOCTOR OF PHILOSOPHY

by
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to the

DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR
SEPTEMBER, 1984

CERTIFICATE

Certified that the work presented in this thesis entitled, 'SOME STUDIES OF DYNAMICAL SYMMETRY BREAKING IN GAUGE MODELS' by Janardan Prasad Singh has been carried out under my supervision and that this has not been submitted elsewhere for a degree.

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ACKNOWLEDGEMENTS

I express my deep gratitude to Professor H.S.Mani for his valuable guidance and keen interest throughout the course of this work. I have learnt a great deal from him.

I am thankful to ProfessorsR.Ramachandran and Tulsi Dass and Dr.S.D.Joglekar from whom I have learnt a lot.

I feel pleasure in thanking many of my colleagues, friends and well-wishers at I.I.T. Kanpur for their interest shown in my research work and for the enjoyable company provided during my stay here. I would especially like to thank Drs.A.Mukherjee and M.Rafat for a critical reading of the manuscript, and to Mr.P.K.Srivastava, Mr.P.K.Raina, Miss Ashu Tyagi, Miss Anuradha Misra and Mr.Manu Mathur for assisting me in proof-reading.

Correspondance with Professors R.Delbourgo and H.Pagels for some clarifications is gratefully acknowledged.

I am greatly indebted to my family members for their forbearance and support in seeing me through this work.

Finally, the patience of Mr.U.S.Misra in careful typing of the manuscript and nice cyclostyling by Mr.L.S.Rathaur and Mr.H.K.Panda is highly appreciated.

-J.P. SINGH

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Thesis entitled, 'Some Studies of Dynamical Symmetry Breaking in Gauge Models' by Janardan Prasad Singh in partial fulfilment of the requirements of the Ph.D. degree of the Department of Physics, Indian Institute of Technology, Kanpur.

The idea of generating the masses of fundamental particles by their self-interactions has drawn a great deal of attention after the development of Weinberg-Salam model of electro-weak interactions. One way to do this is via the "Higgs mechanism". But this procedure is neither theoretically appealing nor is backed by any experimental evidence. Moreover, it creates theoretical obstacles: asymptotic freedom may be easily spoiled, there may be difficulties with quantum gravity, and there is the "fine tuning" problem. Alternative to this is dynamical symmetry breaking (DSB). Here, the asymmetry of the vacuum manifests itself through the non-vanishing vacuum expectation value of a composite operator and the scalar bosons occur as bound states. The mass scale in the originally massless theory is set essentially by the process of renormalization. However, whatever one gains by adopting DSB, one pays for by loosing control on the theory (its solutions) and general ability to perform even approximate calculations. In view of this, it seems imperative to do calculations even in simple models with more rigour.

In the present thesis, we re-examine the Abelian gauge model proposed by Jackiw and Johnson, where a massless

fermion field has a vector and an axial-vector gauge interactions. It has been claimed that DSB is realized in this model. The ratio of the axial-vector-boson mass to the fermion mass ($\frac{\mu}{m}$) has been calculated by different authors employing different approximations. We have investigated this model using the "gauge technique" which is suitable for low--energy calculations such as the present one. In the limit where the ratio of the axial to the vector coupling constants becomes small, or, consistently, in the limit where $\frac{\mu}{m}$ becomes small, we have derived an approximate solution for the fermion spectral function. This, in contrast to the findings of several authors, gives an extremely small value for $\frac{\mu}{m}$.

Non-Abelian gauge theories, being asymptotically free, are better candidates to look for DSB. The strength of the interaction at long ranges in such theories can become very large. Given an attractive channel such forces might produce bound states which could act as dynamical Goldstone particles. Among the realistic cases where DSB has been applied, most successful one is the chiral symmetry breaking (XSB) in QCD. XSB solutions for the quark propagator have been used to calculate various quantities in low-energy hadron physics, such as pion decay constant, pion electromagnetic form factor, amplitude for $\pi^0 \to \gamma\gamma$, etc., and also to resolve U(1) problem in QCD. The results obtained are encouraging.

We further apply these XSB solutions to calculate anomalous magnetic moments (a.m.m.'s) of up- and down-quarks

and their mass difference. These are the modern version of old problems of a.m.m.'s of proton and neutron and their mass difference if one seeks explanation in terms of the constituent quarks.

To calculate a.m.m. of light quarks, we have used XSB solutions of quark propagator of both kinds: solutions based on infrared and asymptotic properties of QCD. Calculations have been done in one gluon exchange approximation using both "singular" and "nonsingular" forms of gluon propagator. We find that "singular" form of gluon propagator in axial gauge, and the XSB solution obtained by using the above propagator and longitudinal part of quark-gluon vertex form a good combination. The quark a.m.m.'s obtained in this way, along with effective quark mass, have been used to calculate proton- and neutron-magnetic moments using regular SU(6) state vectors for baryons. We find a good agreement with experimental results. With certain assumptions, it has also been used to calculate both spin dependent potential between a heavy quark and a light antiquark, and the differential cross-section for $q\bar{q} \rightarrow q\bar{q}$. In these cases, no significant effect of the a.m.m. term over the charge term has been found.

In the gap equation for dynamical quark mass (Σ_D), we introduce QED. Using asymptotic solution for Σ_D for the unperturbed case, we find that to the lowest order in coupling constants the wrong sign of $\Sigma_D^u - \Sigma_D^d$ persists for all reasonable values of momenta.

Our work strengthens the belief that mass generation is unlikely in case of completely massless Abelian gauge

models. Our calculation of a.m.m. of light quarks and some of its phenomenological applications emphasize the role of dynamical quark mass for hadron physics.

Chapter 1

INTRODUCTION

With the recent discoveries[1] of $W^{\pm}$ and Z^{o}, all aspects of the standard $SU_c(3)xSU_L(2)xU(1)$ model seem to be well established with the exception of the Higgs sector. Even if the standard model is integrated in a grand unified model such as SU(5), one is left with an embarrassing number of free parameters — the self couplings and Yukawa couplings of the Higgs bosons — with which to fit the data. In all modern fundamental theories, Higgs fields are introduced solely to effect the breakdown of symmetry (including global chiral symmetry). There is no other compelling theoretical reason for scalar fields (save supersymmetry). Moreover, their presence creates theoretical obstacles: They can easily spoil the asyomptotic freedom that was at the heart of gauge coupling unification. The requirement of the perturbation theory for the weak interaction that there must be at least one physical Higgs boson with $m_H = 0\ (m_W)$, and in any case $\lesssim 0\ (1)$ TeV, is difficult to meet[2]: An elementary scalar particle can acquire large masses ($\delta m_H^2 = 0\ (m_P^2)$ and $O(m_X^2)$ where m_P is the Planck mass $\simeq 10^{19}$ GeV and m_X is the grand unification mass scale $\gtrsim 10^{15}$ GeV) from its propagation through short distance structure of space-time and from its interaction with heavy Higgses (with masses $\sim m_X$), while the radiative corrections will yield $\delta m_H^2 = O(\alpha^n)xO(\Lambda^2)$ which

is difficult to adjust with cut-off $\Lambda \approx m_P$ or m_X.

Alternatively, one may look for the source of spontaneous symmetry breaking (SSB) in the dynamics of the gauge theories themselves. If the asymmetry of the vacuum manifests itself through the non-vanishing vacuum expectation value of a composite operator rather than an elementary scalar field present in the Lagrangian, then this is referred to as dynamical symmetry breaking (DSB). Here the scalar bosons (in particular, Goldstone bosons) occur as bound states. The massless Goldstone bosons may appear as physical particles on account of originally massless fermions acquiring mass (chiral symmetry breaking), or they may combine with the gauge bosons to make them massive (gauge symmetry breaking) exactly like the Higgs case.

Naturally, in such theories there will be fewer free parameters. In fact, if one supposes that the world is described by a grand unified dynamically broken gauge theory, then the single gauge coupling can be traded for the mass scale using dimensional transmutation. The mass scale in the originally completely massless theory is set essentially by the process of renormalization, necessary to remove the divergences present in any renormalizable theory. In such a world all mass ratios and the fine-structure constants are in principle calculable. The price for such an economical choice may be that in practice any calculation worth a

practical significance will be extremely difficult.

The question of whether a given symmetry is broken or not is a dynamical one. In theories with Higgs scalars one can break any symmetry by manipulating the Higgs Lagrangian. DSB provides a self-consistent way to realize SSB.

In case of DSB, there is no mass renormalization because there is no mass to be renormalized. The softness of dynamical mass generation is to be distinguished from the generation of mass from Higgs fields acquiring a vacuum expectation value. In the latter case mass renormalization is required and the fermion mass is not, in general, computable[3]. There is the further suggestion, at least in some approaches to DSB (e.g. in technicolor theories) that a new region $\gtrsim$ 100 GeV of high-energy physics with heavy quarks and leptons must appear.

In DSB, the scale of symmetry breaking is the scale at which the dynamics become strong. There are no dimensionful couplings, as there are in scalar theories, to set the scale of the dynamics. The dimensionless coupling constant of a non-Abelian gauge theory varies with the logarithm of the energy and it may take a huge change of energy before the coupling becomes large. For these reasons DSB is much more attractive in grand unified theories than the usual symmetry breaking by scalar fields[4]. Besides this, DSB figures prominently in the recent surge of interest in supersymmetric theories[5].

The idea of DSB was borrowed originally from the theory of superconductivity[6], where a transverse photon acquires a mass M_γ by "eating up" electron-hole bound state such that the free magnetic field equation $\nabla^2 B = 0$ becomes[7]

$$\nabla^2 B = M_\gamma^2 B \text{ or } B_x \sim e^{-M_\gamma x} \tag{1.1}$$

which explains Meissner damping qualitatively.

There is a simple,intuitive,necessary condition for the occurrence of DSB in a given channel: The forces in that channel must be sufficiently attractive to bind a zero-mass Goldstone boson. In particular, in asymptotically free theories the strength of the interaction at long ranges, or at small momenta, can become very large, irrespective of the value of the "physical" couplings. Given an attractive channel such forces might inevitably produce bound states which could act as dynamical Goldstone bosons[8].

The existence of any bound state, and in particular of a zero-mass bound state, can be examined by a Bethe-Salpeter (BS) equation. The existence of a solution of the BS equation is equivalent to the existence of a symmetry breaking solution to the Schwinger-Dyson (SD) equation of the theory[9]. Usually the existence of the symmetry breaking solutions is not shown to occur necessarily, but rather that the integral equationsof the theory are consistent with such solutions. One assumes here that the nonperturbative

masses depend analytically on the renormalized coupling constants, but that conventional perturbation theory will not reveal this structure. Another possibility is that the mass depends nonanalytically on the renormalized coupling constants[10,11]. The energy gap in a superconductor has similar nonanalytic behavior in the electron-electron coupling[12].

The above method in linearised approximation yields a homogeneous equation which admits trivial solution (no symmetry breaking) as well as the symmetry breaking solution. Hence, one is unable to determine whether SSB does in fact occur.

In another approach, one calculates the effective potential for composite operators, and that enables one to study the stability of the system in a way that closely parallels the traditional treatment with fundamental scalars and to develop a stability criterion for candidate vacua[13]. But as recently shown by Miller[14], an effective potential evaluated perturbatively can possess false minima, artefacts of the approximation itself. In the present thesis, this approach will not be adopted.

Of course, whatever one gains by adopting DSB one pays for by loosing one's control.on the theory (its solutions) and, short of acquiring a considerable amount of insight into nonperturbative effects, one's general ability to perform even approximate calculations. In view of this

fact, it is important to do calculations even in simple models with more rigour. The experience gained from such calculations can be used later for more complicated and realistic cases. In Chapter 2, we shall introduce an early Abelian gauge model proposed by Jackiw and Johnson[15], and then investigate it more rigorously using the so called "gauge technique". In Chapter 3, we review chiral symmetry breaking (XSB) in QCD by deriving the gap equation and finding out its asymptotic solution. XSB solution has also been found out in infrared region through the SD equation using "singular" form of the gluon propagator. We illustrate the use of dynamical quark mass by showing how pion decay constant is calculated[15,16]. In Chapter 4, we use XSB solutions obtained by various authors to calculate anomalous magnetic moments (a.m. m.'s) of light quarks. We use solutions of both kinds : solutions based on infrared and asymptotic properties of QCD. We also work out some phenomenological implications of the presence of a.m.m. In Chapter 5, we calculate u-d quark mass difference and then comment on possible reasons for getting wrong sign of the mass difference. Finally, Chapter 6 contains some concluding remarks. Some of the mathematical details have been put in Appendices.

Chapter 2

A MODEL EXAMPLE

2.1 Introduction

DSB in particle physics was initiated by the Nambu-Jona-Lasinio model[9] based on four-Fermi interaction where it was shown that a theory may possess a XSB bound-state Goldstone boson. Next Schwinger[17] made the crucial observation that it is possible to have a pole in the polarization tensor of a vector gauge boson at zero momentum transfer. This pole leads to SSB by giving a mass to the otherwise massless gauge particle, a process known as the Schwinger mechanism. It turns out that the Schwinger mechanism is unique for 1+1 dimensional QED[18] and it occurs due to topological reasons[4]. Another argument why the Schwinger mechanism does not work for, say, 3+1 dimensional QED is due to charge-conjugation and T-invariance[18]. If charge-conjugation is unbroken, a vector field with odd charge-conjugation can not absorb a Goldstone boson which is a bound-state of fermion-antifermion pair in the scalar state having even charge-conjugation. A similar argument works for T-invariance also. By contrast, an axial-vector gauge field is C-even and T-odd as is the bound pseudoscalar state of fermion-antifermion pair. So axial-vector and mixed parity gauge fields can acquire mass in this way.

But a pure axial-vector exchange is repulsive and can not form a bound-state. On the other hand if the theory contains both vector as well as axial-vector gauge particles and the vector coupling is stronger than the axial coupling so that the net attractive force can form a bound-state Goldstone boson, then one can hope that the Schwinger mechanism will work for the axial-vector gauge boson. The Jackiw-Johnson[15] (JJ) model implements this idea in an Abelian gauge model. It has been claimed that in this model both chiral symmetry as well as axial-vector gauge symmetry get broken[15,18-20]. However, different authors have employed different approximations and consequently have obtained different results for the ratio of the axial-vector-boson mass to the fermion mass. This is one of the reasons why we took up this model. What we are going to describe in the rest of this Chapter is contained in Reference [21].

2.2 Review of Earlier Work

The JJ model is described by a zero bare-mass Lagrangian given by

$$\mathcal{L} = \bar{\Psi}\,(i\not\partial - g \not A - g' \not B \gamma_5)\Psi - 1/4(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - 1/4(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 \qquad (2.1)$$

which is gauge invariant under the full $U_V(1) x U_A(1)$ set of transformations, namely the ordinary phase transformations

$$\Psi(x) \to \exp(i\lambda(x))\Psi(x), \quad A_\mu \to A_\mu - \partial_\mu \lambda(x)/g$$

as well as the chiral set

$$\Psi(x) \to \exp(i\lambda'(x)\gamma_5)\ \Psi(x), B_\mu \to B_\mu - \partial_\mu \lambda'(x)/g'.$$

In searching for the phase which breaks chiral symmetry, namely $\langle\bar{\Psi}\Psi\rangle \neq 0$, one assumes that the fermion acquires a non-zero (renormalized) mass and correspondingly that the inverse fermion propagator $S^{-1}(p)$ develops a non-$\not p$ component. This will imply, via the axial Ward-Takahashi (WT) identity, that the axial-vector vertex $\Gamma_{\mu 5}$ has a pole which is associated with the induced Goldstone excitation. As a result the axial self-energy develops a pole in its momentum variable. This in turn implies[15] a non-zero axial-vector-meson mass μ. Therefore, in such a chiral model, one can anticipate a relation between μ/m and the axial-meson coupling constant g', where m is the dynamically generated fermion mass. Furthermore, it is also clear that μ should tend to zero as $g' \to 0$ (that is, in the free-field-theory limit).

As emphasized by Delbourgo and Keck[19] (DK), the gauge technique[22] may be a suitable tool for investigating such problems since the gauge identities must be appropriately respected whether or not DSB ensues. Secondly, DSB is a low-energy phenomenon which occurs due to bound-state formation, and it is precisely this region of energy where the gauge

technique works very well[23,24]. Below, we shall briefly review the work of DK and fix up our notation.

We have vector as well as axial WT identities for the renormalized Green functions

$$k^{\mu} S(p) \Gamma_{\mu} (p,p-k)\ S(p-k) = S(p-k) - S(p) \tag{2.2a}$$

$$k^{\mu} S(p) \Gamma_{\mu 5}(p,p-k)\ S(p-k) = \gamma_5\ S(p-k) + S(p)\gamma_5 \ . \tag{2.2b}$$

Here S denotes the complete fermion propagator, whose unrenormalized form is given by

$$\begin{aligned} iS(p)_{\beta\alpha} &= \int d^4 y\ e^{ip.(x-y)}\ i\ S_{\beta\alpha}\ (x-y) \\ &= \int d^4 y\ e^{ip.(x-y)} \langle 0 | T(\Psi_{\beta}(x)\ \bar{\Psi}_{\alpha}\ (y)) | 0 \rangle . \end{aligned} \tag{2.2c}$$

Γ's stand for the irreducible vertex functions with coupling constants factorized out. Their unrenormalized forms are defined by

$$\begin{aligned} iS(p)\ \Gamma_{\mu}(p,p')\ i\ S(p') = \int d^4x\ d^4y\ e^{ipx}\ e^{-ip'y} \langle 0 | T \\ (\Psi(x)\ J_{\mu}(o)\ \bar{\Psi}(y)) | 0 \rangle \end{aligned} \tag{2.2d}$$

and

$$\begin{aligned} iS(p)\ \Gamma_{\mu 5}(p,p')\ iS(p') = \int d^4x\ d^4y\ e^{ipx}\ e^{-ip'\ y} \langle 0 | T \\ (\Psi(x)\ J_{\mu 5}(o)\ \ \bar{\Psi}\ (y)) | 0 \rangle , \end{aligned} \tag{2.2e}$$

where

$$J_{\mu}\ (x) = \bar{\Psi}\ (x)\gamma_{\mu}\ \Psi\ (x) \tag{2.2f}$$

and

$$J_{\mu 5}(x) = \bar{\Psi}(x)\gamma_\mu \gamma_5 \Psi(x) . \tag{2.2g}$$

Since Equation (2.2b) has been derived in a formal way on the assumption of axial current conservation, one has to ensure that quantum loop corrections do not invalidate it. This is possible only if regularization anomalies are absent or else cancel—in practice this may mean a doubling of fermion fields. We have further assumed that Equation (2.2b), valid for unrenormalized Green's functions, retains its form after a multiplicative renormalization.

If we write the fermion propagator in terms of a spectral function as

$$S(p) = \left(\int_{-\infty}^{-m} + \int_{m}^{\infty} \right) \frac{\rho(W)\, dW}{\not{p} - W + i\varepsilon'\varepsilon(W)} \tag{2.3}$$

where $\rho(W)$ is the fermion spectral function (Appendix A), then, up to a transverse correction, an appropriate solution of Equations (2.2a) and (2.2b) can be written as[19]

$$S(p)\Gamma_\mu(p,p-k)\, S(p-k) = \int \frac{dW \rho(W)}{\not{p} - W} \gamma_\mu \frac{1}{\not{p} - \not{k} - W} \tag{2.4a}$$

$$S(p)\Gamma_{\mu 5}(p,p-k)\, S(p-k) = \int \frac{dW \rho(W)}{\not{p} - W} \left(\gamma_\mu\gamma_5 - \frac{2\, W\, k_\mu \gamma_5}{k^2}\right) \frac{1}{\not{p}-\not{k}-W} . \tag{2.4b}$$

The appearance of the pole term at the axial leg indicates the presence of the pseudoscalar Goldstone mode. This induces a dynamical singularity in the meson self-energy

$$\pi_{\mu 5 \nu 5}(k) = ig'^2\, Z_2 \int (dp)\, \mathrm{Tr}(S(p)\, \Gamma_{\mu 5}(p,p-k)\, S(p-k)\, \gamma_\nu \gamma_5);\ (dp) = \frac{d^4p}{(2\pi)^4} \tag{2.5}$$

as $k \to 0$ and this, in turn implies that the original zero-mass axial meson acquires a finite mass μ . Here Z_2 is the fermion wave function renormalization constant. Substituting Equation (2.4b) in Equation (2.5) we have

$$\pi_{\mu 5 \nu 5}\ (k) = \pi_{\mu\nu}(k) + \left(\frac{k_\mu k_\nu}{k^2} - g_{\mu\nu}\right) 8\ ig'^2\ Z_2 \int\!\!\int \frac{(dp)\ dW\ W^2 \rho\ (W)}{(p^2-W^2)[(p-k)^2-W^2]} \tag{2.6}$$

where $\pi_{\mu\nu}$ is the conventional QED self-energy for a coupling g'. Invoking conservation of axial current, this can be written as

$$\pi_{\mu 5 \nu 5}\ (k) = (-k^2 g_{\mu\nu} + k_\mu k_\nu)\ \pi_A\ (k^2); \tag{2.7}$$

then clearly

$$\pi_A\ (k^2) \to -g'^2\ \lambda^2/k^2 \quad \text{as } k^2 \to 0 \tag{2.8}$$

by virtue of the Goldstone pole in $\Gamma_{\mu 5}$. Here

$$\lambda^2 = -8\, i\, Z_2 \iint \frac{(dp)\, dW\, W^2 \rho\, (W)}{(p^2 - W^2)^2} \,. \tag{2.9}$$

Now it can be seen that the renormalized inverse propagator (Z_3 being the axial-meson wave function renormalization constant)

$$D^{-1}_{\mu 5 \nu 5}(k) = (-g_{\mu\nu} + k_\mu k_\nu / k^2)(k^2 Z_3 + k^2\, \pi_A(k^2)) + \text{a gauge fixing term} \tag{2.10}$$

no longer has a zero at $k^2 = 0$. It is shifted elsewhere, to the physical $(\text{mass})^2 \mu^2$ which can be determined by invoking the renormalization condition $D^{-1} \simeq (k^2 - \mu^2)$ near $k^2 = \mu^2$. In the pole approximation

$$\mu^2 = g'^2 \lambda^2 = -8\, i\, g'^2\, Z_2 \iint \frac{(dp)\, dW\, W^2 \rho\, (W)}{(p^2 - W^2)^2} \,. \tag{2.11a}$$

After Wick rotation ($p_o \to ip_4$, $p^2 = \sum_{i=1}^{4} p_i^2$) and introducing the dimensionless quantities

$$\omega = W/m, \; r(\omega) = m\rho(W), \; q^2 = p^2/m^2,$$

Equation (2.11a) can be written, after the angular integration, in the form

$$\frac{\mu^2}{m^2} = \frac{g'^2\, Z_2}{2\,\pi^2} \iint \frac{\omega^2 r(\omega)\, q^2 d\omega dq^2}{(q^2 + \omega^2)^2} \tag{2.11b}$$

To determine $r(\omega)$ or $\rho(W)$, one writes the Schwinger-Dyson (SD) equation for the fermion propagator:

$$Z_2^{-1} = S(p)\,\not{p} - ig^2 \int (dk)\, S(p) \Gamma_\mu (p,p-k)\, S(p-k)\, D_A^{\mu\nu}(k) \gamma_\nu$$

$$-ig'^2 \int (dk)\, S(p)\, \Gamma_{\mu 5}(p,p-k) S\,(p-k)\, D_B^{\mu\nu}(k) \gamma_\nu \gamma_5 \,. \tag{2.12}$$

The complete gauge boson propagators in Eq.(2.12) are replaced by the bare ones, which in the Landau gauge can be written as

$$D_{A,B}^{\mu\nu}(k) = \frac{-g_{\mu\nu} + k_\mu k_\nu/(k^2+i\varepsilon)}{k^2 - \mu_{A,B}^2 + i\varepsilon} \qquad \mu_A = 0,\ \mu_B = \mu. \tag{2.13}$$

Then using Equations (2.4a) and (2.4b) in Equation (2.12) and the fact that (Appendix A)

$$Z_2^{-1} = \int \rho(W)\, dW$$

one arrives at the following self-consistent equation for $\rho(W)$ (derived in Appendix A):

$$-\varepsilon(W)\rho(W)(W-m) = \int \frac{dW'\, \rho(W')}{\pi(W-W')}\ \mathrm{Im}\,(\Sigma_A(W, W') + \Sigma_B(W,W')) \tag{2.14}$$

where (see Appendix B):

$$\mathrm{Im}\,\Sigma_A(W,W') = \frac{3\, g^2 W'}{16\,\pi W^2} (W^2-W'^2)\theta(W^2-W'^2) \tag{2.14a}$$

and

$$\mathrm{Im}\,\Sigma_B(W,W') = \frac{-g'^2}{32\pi W^3}\left((W^2+6WW'+W'^2-2\mu^2)\phi_\mu + \frac{(W^2-W'^2)^2}{\mu^2}(\phi_\mu - \phi_o)\right) \tag{2.14b}$$

$$\phi_\mu = \{[W^2-(W'+\mu)^2][W^2-(W'-\mu)^2]\}^{1/2}\ \theta[W^2-(W'+\mu)^2].$$

In view of the fact that the integral equation (2.14) is intractable, DK resorted to an assumed form* of $\rho(W)$:

$$m\rho(W) = \varepsilon(W)\ \frac{(W^2/m^2-1)^{-1-2\delta}2^{2\delta}\Gamma(-\delta-\delta')}{\Gamma(-\delta+\delta')\Gamma(-2\delta)}\left[\frac{m}{W}F(-\delta-\delta',-\delta-\delta';-2\delta;1-\frac{W^2}{m^2}) + \left(\frac{\delta+\delta'}{\delta-\delta'}\right)F(-\delta-\delta',1-\delta-\delta';-2\delta;1-\frac{W^2}{m^2})\right]$$

with $\delta = 3g^2/16\pi^2$, $\delta' = 3g'^2/16\pi^2$. This guessed form of $\rho(W)$ satisfies the following requirements: (i) it has the correct infrared and ultraviolet behavior and (ii) it reduces to the correct QED form in the limit $g' \to 0$. Their guessed form of $\rho(W)$ gives the following result:

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* In a later paper Delbourgo and Kenny[25] claim to have made a better guess for $\rho(W)$ which we have exhibited later in this Chapter. But the result for μ/m comes out to be the same in both cases.

$$\mu/m \simeq \frac{4}{\sqrt{3}} \quad \frac{g'^2}{g^2 - g'^2} \, .$$

2.3 Approximate Solution of $\rho(W)$ and Equation for the Mass Ratio

Let us consider a limiting case when $g' << g$. Then DK's result for μ/m is $O(g'^2/g^2)$ whereas JJ obtained

$$\mu^2/m^2 \simeq \frac{4}{3} \quad \frac{g'^2}{g^2 - g'^2} \, ,$$

which, in the above limit, will give μ/m to be $O(g'/g)$. Thus we can attempt to obtain the value of μ/m in that limit since we can hope to solve the integral equation in that unit: we shall employ a solution obtained by (non-perturbative) gauge technique in the gauge coupling g and by a perturbation expansion in g'. We will look for a consistent solution with $\mu/m << 1$. Needless to say, the $\rho(W)$ determined in this way should satisfy the requirements stated in Section 2.

In the limit stated above, one can attempt an expansion of $\rho(W)$ in a power series in μ/m : $\rho(W) = \rho^o(W) + \Delta\rho(W)$. Here $\rho^o(W)$ is the fermion spectral function with $\mu = 0$ and $\Delta\rho(W) = (\mu/m)\,\rho^{(1)}(W) + (\mu^2/m^2)\,\rho^{(2)}(W) + \ldots\ldots$
It can be easily checked, as will be shown later, that $\rho^o(W)$ will have the same form as found in pure QED[22] with the replacement $g^2 \to g^2 - g'^2$. When this $\rho^o(W)$ is used to calculate the axial-meson mass, one gets a zero answer

(shown in Appendix C) as one should, since the Goldstone mode does not materialise in QED[19]. It has been shown in References [23] and [24] that the fermion (interacting with a massless gauge boson) spectral function determined from the longitudinal vertex is sufficient to determine the infrared properties of the theory; hence the correction to $\rho^{0}(W)$ coming from inclusion of the transverse vertex would not give a non-zero value for μ. Thus the gauge technique tells us that in all calculations where the axial-meson propagator with $\mu = 0$ has been used to calculate the fermion mass ($m \neq 0$), and subsequently the axial-meson mass itself, one should get $\mu = 0$. Thus it is only $\Delta\rho(W)$ which can give a non-zero contribution, if any, to μ. Our attempt will be confined to calculate only the first term in the expansion of $\Delta\rho(W)$. Since the coefficient μ/m of $\rho^{(1)}(W)$ comes from an internal meson propagator, $\rho^{(1)}$ must have an overall factor of g'^2 as we shall show later, i.e., $\Delta\rho(W) \propto (\mu/m)g'^2$ in the lowest approximation. If this $\Delta\rho(W)$ is substituted in equation (2.11b), then one can see that the expression obtained for μ/m in this way will have a factor of g'^4. This is our main result. Below we shall give its derivation leaving some of the mathematical details for the Appendices.

We shall solve Equation (2.14) for $g'^2/g^2 << 1$, or, consistently, for $\mu/m << 1$. Our approximation consists of simplifying the expression for $\mathrm{Im}\,\Sigma_B$ and then solving for $\rho(W)$ perturbatively in the small parameters stated above.

Let us define $3g^2/16\pi^2 = \delta$, $3g'^2/16\pi^2 = \delta'$, $\delta - \delta' = \delta_2$, and break $r(\omega)$ into even and odd parts as

$$r(\omega) = r_+(\omega) + r_-(\omega) \tag{2.15}$$

where

$$r_+(\omega) = \frac{1}{2}\{r(\omega) + r(-\omega)\}, \quad r_-(\omega) = \frac{1}{2}\{r(\omega) - r(-\omega)\}. \tag{2.15'}$$

Now, in the infrared region, where $(|W| - m) \lesssim \mu$, Equation (2.14) can be written as

$$\omega^2 \, \varepsilon(\omega)(\omega-1)r(\omega) = -\int \frac{d\omega' \; r(\omega')}{\omega - \omega'} \left(\delta\omega'(\omega^2-\omega'^2) + \delta' \frac{(\omega^2-\omega'^2)^3}{6\,\omega\mu^2/m^2}\right) \theta(\omega^2-\omega'^2). \tag{2.16}$$

The solution of Equation (2.16) with the last term dropped is known[23,26] (derived in Appendix C)

$$r^{o\prime}(\omega) = \frac{\varepsilon(\omega)(\omega^2-1)^{-1-2\delta}}{\Gamma(-2\delta)\; 2^{-2\delta}} \left(\omega^{-1} F(-\delta, -\delta; -2\delta; 1-\omega^2) + F(-\delta, 1-\delta; -2\delta; 1-\omega^2)\right). \tag{2.17}$$

On writing

$$r_{\pm}(\omega) = r^{o\prime}_{\pm}(\omega) + \Delta r'_{\pm}(\omega) \tag{2.18}$$

one finds (shown in Appendix D) that

$$\Delta r'_{\pm}(\omega) \simeq -\frac{\delta'}{3}\,\frac{m^2}{\mu^2}\,\frac{\Gamma(-2\delta)}{\Gamma(2-2\delta)}\,(\omega^2-1)^2\, r^{o\prime}_{\pm}(\omega). \tag{2.19}$$

In a region where W is such that $\mu/(W-m) << 1$ and remembering that in Equation (2.14), $\mathrm{Im}\,\Sigma_B\,(W,W')$ is multiplied with the weight function $\rho(W')$, which is concentrated mainly at $W'=m$, Equation (2.14) can be written as (Appendix E)

$$\omega^2\varepsilon(\omega)\ (\omega-1)\ r(\omega) \simeq -\int \frac{d\omega'\ \ r(\omega')}{\omega-\omega'}\Big[\ \delta_2\omega'(\omega^2-\omega'^2)\theta(\omega^2-\omega'^2)-\delta'$$

$$x(\omega'(\omega^2-\omega'^2)+\frac{(\omega^2-\omega'^2)^3}{6\omega\mu^2/m^2})(\theta[\omega^2-(\omega'+\mu/m)^2]-\theta(\omega^2-\omega'^2))\Big]. \tag{2.20}$$

The solution of Equation (2.20) with $\mu/m = 0$ is easily found out to be

$$r^0(\omega) = c\ \frac{\varepsilon(\omega)(\omega^2-1)^{-1-2\delta_2}}{\Gamma(-2\delta_2)2^{-2\delta_2}}\ (\omega^{-1}\ F(-\delta_2,\ -\delta_2;-2\delta_2;1-\omega^2)$$

$$+F(-\delta_2,1-\delta_2;-2\delta_2;\ 1-\omega^2)), \tag{2.21}$$

where c is a constant which may depend on δ'. Again writing

$$r(\omega) = r^0(\omega)+\Delta r(\omega) \tag{2.22}$$

one finds that (shown in Appendix D)

$$\Delta\ r_+(\omega)\simeq -\frac{44}{9}\,\delta'(\mu/m)(\omega^2-1)^{-1}\ r_+^0(\omega) \qquad \lambda^2 \lesssim \omega^2 \lesssim \infty \tag{2.23a}$$

$$\Delta\, r_-(\omega)\simeq -\frac{44}{9}\ \delta'(\mu/m)(\omega^2-1)^{-1}\,\omega r_+^0(\omega) \qquad \lambda^2 \lesssim \omega^2 \lesssim \infty \tag{2.23b}$$

where λ is defined in such a way that $(\lambda-1) \ll 1$ and $\mu/m \ll (\lambda-1)$ (one can think of $\mu/m = O[(\lambda-1)^2]$) .

We have one unknown constant c which could be determined by the condition that $r(\omega)$ should be continuous at $\omega = \pm(1+\mu/m)$. Unfortunately $\Delta r(\omega)$ is not known for $|\omega|$ just above $1+\mu/m$. However, it is clear from Equation (2.17) that $c = 1 + O(\delta')$. Let us compare our result for $r_+(\omega)$ with that obtained by Delbourgo and Kenny[25] (translated into our notation):

$$r_+(\omega) \simeq \varepsilon(\omega)\omega^{-1} \left(\frac{(\omega^2-1)^{-1-2\delta}}{\Gamma(-2\delta)2^{-2\delta}} F(-\delta, -\delta; -2\delta; 1-\omega^2) \right.$$

$$\left. + 2\delta' F(1+\delta_2, 1+\delta_2; 1; 1-\omega^2) \right).$$

In Table 1 we have tabulated both the results for $\varepsilon(\omega)\omega\, r_+(\omega)$ for different values of ω^2. It is to be noted that in the calculation of $(\mu/m)^2$, it is only the second term in Delbourgo and Kenny's expression for $r_+(\omega)$ which contributes, and in our case it is $\Delta r_+'(\omega)$ and $\Delta r_+(\omega)$, given by Equations (2.19) and (2.23a) respectively, which contribute. These two expressions are different from each other. In our opinion our approximate form of $\rho(W)$ is more reliable, in the region $g'^2/g^2 \ll 1$ and $\mu/m \ll 1$ as it is based on an explicit calculation.

Table 1

Comparison of our result for $\varepsilon(\omega)\,\omega\, r_+(\omega)$ with that obtained by Delbourgo and Kenny for different values of ω^2.

	Our result	Delbourgo and Kenny's result
(i) $\omega^2 \to 1$	$\frac{(\omega^2-1)^{-1-2\delta}}{\Gamma(-2\delta)2^{-2\delta}}\,[1+\frac{1}{2}\delta(\omega^2-1)-\frac{\delta}{4}\frac{(1-\delta)^2}{1-2\delta}(\omega^2-1)^2+\ldots$	$\frac{(\omega^2-1)^{-1-2\delta}}{\Gamma(-2\delta)2^{-2\delta}}[1+\frac{1}{2}\delta(\omega^2-1)-\frac{\delta}{4}\frac{(1-\delta)^2}{1-2\delta}\times$
	$-\frac{\delta'}{3}\frac{\Gamma(-2\delta)}{\Gamma(2-2\delta)}\frac{(\omega^2-1)^2}{\mu^2/m^2}+\ldots]$	$(\omega^2-1)^2+\ldots]+2\delta'[1-(1+\delta_2)^2(\omega^2-1)+\ldots]$
(ii) $\omega^2 \simeq \lambda^2$	$c\,\frac{(\omega^2-1)^{-1-2\delta_2}}{\Gamma(-2\delta_2)2^{-2\delta_2}}[1+\frac{1}{2}\delta_2(\omega^2-1)-\frac{\delta_2}{4}\frac{(1-\delta_2)^2}{1-2\delta_2}(\omega^2-1)^2+\ldots$	$\frac{(\omega^2-1)^{-1-2\delta}}{\Gamma(-2\delta)2^{-2\delta}}[1+\frac{1}{2}\delta(\omega^2-1)-\frac{\delta}{4}\frac{(1-\delta)^2}{1-2\delta}\times$
	$-\frac{44}{9}\delta'\frac{\mu/m}{\omega^2-1}[1+\frac{1}{2}\delta_2(\omega^2-1)+\ldots]]$	$(\omega^2-1)^2+\ldots]+2\delta'[1-(1+\delta_2)^2(\omega^2-1)+\ldots]$
(iii) $\omega^2 \to \infty$	$c\,\frac{(\omega^2)^{-1-\delta_2}}{\Gamma^2(-\delta_2)2^{-2\delta_2}}\ln\omega^2+\ldots$	$\frac{(\omega^2)^{-1-\delta}}{\Gamma^2(-\delta)2^{-2\delta}}\ln\omega^2+2\delta'\frac{(\omega^2)^{-1-\delta_2}}{\Gamma(-\delta_2)\Gamma(1+\delta_2)}\ln\omega^2+\ldots$

To calculate the mass ratio, as given by Equation (2.11b), we have to perform the following integration:

$$\int\int \frac{\omega^2 r(\omega) q^2 d\omega dq^2}{(q^2+\omega^2)^2} = \int\int_{|\omega|=1}^{1+\mu/m} \frac{\omega^2 \Delta r_+'(\omega)\ q^2 d\omega\, dq^2}{(q^2+\omega^2)^2}$$

$$+ \int\int_{|\omega|=1+\mu/m}^{\infty} \frac{\omega^2 \Delta r_+(\omega)\ q^2 d\omega dq^2}{(q^2+\omega^2)^2}$$

First consider the integration over ω :

$$\int \frac{\omega^2 r(\omega) d\omega}{(q^2+\omega^2)^2} = \left(\int_{-(1+\mu/m)}^{-1} + \int_{1}^{1+\mu/m}\right) \frac{\omega^2 \Delta r_+'(\omega)\ d\omega}{(q^2+\omega^2)^2}$$

$$+\left[\left(\int_{-\lambda}^{-(1+\mu/m)} + \int_{1+\mu/m}^{\lambda}\right) + \left(\int_{-\infty}^{-\lambda} + \int_{\lambda}^{\infty}\right)\right] \frac{\omega^2 \Delta r_+(\omega)\ d\omega}{(q^2+\omega^2)^2} . \tag{2.24}$$

The third term gives

$$\left(\int_{-\infty}^{-\lambda} + \int_{\lambda}^{\infty}\right) d\omega \frac{\omega^2 \Delta r_+(\omega)}{(q^2+\omega^2)^2} = -c\,\frac{44}{9}\,\frac{\delta'(\mu/m)2^{2\delta_2}}{\Gamma(-2\delta_2)}\,\frac{1}{(q^2+1)^2}$$

$$\times\left(\frac{(\lambda^2-1)^{-1-2\delta_2}}{1+2\delta_2} + \frac{(\lambda^2-1)^{-2\delta_2}}{4} - \frac{1}{\delta_2}\,\frac{(\lambda^2-1)^{-2\delta_2}}{q^2+1}\right.$$

$$\left. + \text{ terms of } O[(\lambda^2-1)^{1-2\delta_2}]\right)$$

$$-c\,\frac{44}{9}\,\frac{\delta'(\mu/m)2^{2\delta_2}}{\Gamma(-2\delta_2)}\;\frac{1}{(q^2+1)^2}\;\Big(\frac{\Gamma^2(1+\delta_2)\Gamma(-2\delta_2)\Gamma(-1-2\delta_2)}{\Gamma^2(-\delta_2)}$$

$$-\Gamma(-2\delta_2)\Gamma^2(1+\delta_2)\,F(1+\delta_2,\;1+\delta_2;\;1;\;-q^2)-(q^2+1)\Gamma(-2\delta_2)$$

$$\times\Gamma^2(2+\delta_2)\,F(2+\delta_2,\;2+\delta_2;\;2;\;-q^2)\Big). \qquad (2.25)$$

In the first term of Equation (2,24) the contribution to the integral comes only from the limit $\omega = \pm(1+\mu/m)$*. $\Delta r_+(\omega)$ in the range of the second integral is not known. However, if $\Delta r_+(\omega)$ is smooth enough at $\omega = \pm(1+\mu/m)$ so that the integral is continuous at this point, then the whole contribution to Equation (2,24) comes from the second term of Equation (2.25); if the integral is not continuous at $\omega = \pm(1+\mu/m)$ then μ^2/m^2 will be logarithmically divergent. Assuming that the first possibility is realized, we get

$$\frac{\mu}{m} \simeq c\,\frac{352}{27}\,\delta'^2\Big(\frac{\Gamma^2(1+\delta_2)}{\Gamma(2+2\delta_2)}-\frac{\delta_2}{2(1+2\delta_2)}\,\frac{\Gamma(1-2\delta_2)}{\Gamma^2(1-\delta_2)}\ln\Lambda^2\Big) \qquad (2.26)$$

where Λ is an ultraviolet cut-off. But in a truly consistent

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*The behaviour of integrals involving terms like $r^0(\omega)$ in the infrared region has been discussed by Atkinson and Slim[27].

theory of DSB μ/m should be finite[15*]. On the other hand, we observe that the logarithmically divergent term is smaller by a factor of δ_2 compared with the first term. It is at this order of magnitude in coupling constants that the contribution of the transverse correction to $\Delta r_+(\omega)$ becomes important if we assume that this contribution is non-zero in the (infrared) region of interest. If μ/m has to be finite then the divergent term in Equation (2.26) must cancel the term obtained from the transverse corrections to $\Delta r_+(\omega)$ and we would get

$$\mu/m = O(\delta'^2) .$$

2.4 Conclusion

Our result for the fermion spectral function agrees, by and large, with that obtained by Delbourgo and Kenny[25] in the infrared and ultraviolet regions, but in the intermediate region it disagrees with their result. This implies, in particular that we get the same asymptotic form of the fermion self-energy function as obtained by JJ[19,24]. But

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* The consistency of our approach to this problem requires μ/m to be not only finite but small.

our result for μ/m, in sharp contrast to the findings of earlier authors, is extremely small.

The above result could be an indication that the dynamical gauge symmetry breaking hypothesis is questionable in the JJ model. Indeed using renormalization group arguments Gross[11] has shown that in case of infrared free theories, as the interaction (one or more) responsible for the DSB is removed the masses increase. This, in particular, means that for an elementary particle mass one will not have a smooth transition to the free field theory limit. Gross regards this as an indication that infrared free theories can not produce DSB.

Stern[28], again using renormalization group arguments, has also shown that in completely massless Abelian gauge models, like the JJ model, dynamical mass generation is not possible, at least for weak couplings. However, Stern points out, unlike the case of the gauge boson, fermion mass generation can occur at a nontrivial infrared-stable fixed point.

We have worked in the Landau gauge because in this gauge expressions for Im Σ_A and Im Σ_B are simple. In other gauges, it is difficult to find the solution for fermion spectral function even in QED[26]. Like earlier authors, we have not shown the gauge independence of our result.

Since symmetry breaking in realistic physical models is expected to be an infrared-dominated phenomenon[4], we will now onwards concentrate on non-Abelian gauge models.

Chapter 3

CHIRAL SYMMETRY BREAKING IN QCD AND ITS APPLICATIONS

3.1 Introduction

It is becoming increasingly clear that in QCD there are two physically meaningful concepts of mass: the perturbative or current quark mass m which occurs in the bare QCD Lagrangian (presumably due to weak interactions) and the nonperturbative or dynamically generated quark mass m_{dyn} which dominates low energy hadron physics. To clarify it, consider the QCD Lagrangian with $m=0$; then it has $SU_L(N_F) \times SU_R(N_F)$ symmetry (N_F = number of flavors). The success of PCAC (partial conservation of the axial current) in conjunction with current algebra is an experimental fact. If we assume QCD describes strong interactions, this requires that the above symmetry is dynamically broken down to $SU_V(N_F)$: $\dot{Q}_5^a = 0$, $Q_5^a|0\rangle \neq 0$, $a = 1,2,..,N_F^2 -1$, with N_F^2-1 attendant massless pseudoscalar bosons and $\dot{Q}^a = 0$, $Q^a|0\rangle = 0$. This Nambu-Goldstone (NG) realization of chiral symmetry[29] is the best place where a dynamically broken symmetry seems to be relevant for the real world. When chiral symmetry is explicitly broken by current quark mass ($m \neq 0$) terms, the NG bosons become massive.

To give an idea of why chiral symmetry is realized in the NG way, let us again use the analogy with superconductors,

The Cooper-pairing rearranges the surface of the Fermi sphere because electrons have attractive interaction due to phonon exchange. In a similar way, the attraction in the scalar $q\bar{q}$ channel is so great that it destroys the well-defined Dirac sea and produces, a finite gap (= effective mass) between negative and positive energy states. Another way to express this is to say that a quark condensate is developing:

$$\langle 0 | \bar{\Psi}(x)\, \Psi(x) | 0 \rangle \neq 0, \tag{3.1}$$

also being analogous to the case in superconductors. To give a numerical estimate, it is known[30] that

$$\begin{aligned} \langle 0 | \bar{\Psi}\Psi | 0 \rangle &\simeq -1.6\times 10^{-2}\ \mathrm{GeV}^3 \\ \langle 0 | (g\, G^a_{\mu\nu})^2 | 0 \rangle &\simeq 0.5\ \mathrm{GeV}^4, \end{aligned} \tag{3.2}$$

where Ψ is a quark field, and $G^a_{\mu\nu}$ is the gluon field strength.

In the literature, several "causes" have been put forward for dynamical chiral symmetry breaking (XSB) in QCD. We mention a few of them.

(i) Computer simulations in lattice gauge theory suggest that the chiral symmetry breaks down spontaneously[31], and that the range of force responsible for XSB is relatively short, independent of confinement[32].

Let us recall a crude but basic idea about how XSB occurs in gauge theories. Consider a bound state of a pair of massless $q\bar{q}$. Because of the uncertainty principle, the energy of the ground state (pion) will be given by $E_\pi \approx p - g^2/r \approx p(1-g^2)$ where p and r denote the relative momentum and coordinate, respectively. In a fully relativistic formulation, this relation may be replaced by $E_\pi^2 \approx p^2 - g^4/r^2 \approx p^2(1-g^4)$. When the gauge coupling constant exceeds order one, there will be tachyon bound states, indicating instability of the vacuum. In order to cure this instability, the vacuum rearranges itself and gives mass to quark so as to keep the pion massless. Hence, in the ladder approximation (or, more precisely, in the single gluon exchange approximation), XSB is expected to occur for coupling constants above a certain critical value. In fact a simple analysis of distance dependence of the coupling constant $[\alpha_s(r) = \dfrac{2\pi}{b \ln \frac{1}{r\Lambda}}]$ shows that $\alpha_s(r) \approx 1$ at a distance of about $R \approx 1$ fm which is roughly the size of the hadrons. This sets the scale of non-perturbative confining effects whereas other non-perturbative phenomena (such as XSB) start essentially already at distances of about 0.1 - 0.4 fm (depending on the particular quantity under consideration), where perturbation series still look good enough[33].

(ii) Several authors[16,34-37] have argued that XSB will occur, in any theory of massless fermions interacting through non-Abelian confining gauge fields. Strong coupling lattice treatments precisely seem to demonstrate at the same time XSB and confinement[38]. It has been proved, within the mean field approximation, that confinement and XSB are appearing or disappearing together[39].

From a study of various models, Cornwall[16] has illustrated how a theory of confinement automatically leads to XSB. In many such theories, the quark propagator is expected to be an entire function of the momentum, a characteristic criterion for confinement. A study of the SD equation for the quark propagator, in a confining QCD, by Ball and Zachariasen[35] reveals that, in addition to a chiral-symmetry preserving solution, there exists a broken symmetry solution which has no singularity, thus hinting at confiment. More recently, Acharya and Narayana-Swamy[36], by an examination of WT identities and SD equations in QCD, were lead to the conclusion that there exists a connection between the NG realization of chiral symmetry and quark confinement. In fact, these authors have also demonstrated[40] that chiral symmetry remains unbroken due to single gluon exchange when the gluon propagator has the behavior $D_{\mu\nu} \sim q^{-2}$ as $q^2 \to 0$ (representing nonconfining force). A similar connection between

confinement and XSB has been shown in Reference [37].

(iii) Some authors have demonstrated that the instanton structure of QCD may break the chiral symmetry. The reason for this is, as proposed by Callan, Dashen and Gross[41], that instantons can generate attractive forces between quark and antiquark. Since this topic is not directly relevant for our purpose, we shall not go into its details. It is to be pointed out that the quantum theory of interacting instantons turns out to be rather complicated, so the most interesting effects have not been so far reliably calculated from first principles.

However, in a model dependent approach[42], the instanton parameters, such as the instanton density n_c in space-time and typical radius ρ_c, have been fixed from data (mainly the gluon condensate). From these parameters, the quantitative estimates of $<\bar{\Psi}\Psi>$ and m_{eff} (quark effective mass) have been made. The numbers agree reasonably well with those obtained from other sources (e.g. equation (3.2) for $<\bar{\Psi}\Psi>$).

3.2 Gap Equation and Its Solutions

Here, we shall first derive a generalized gap equation for the mass operator $\Sigma(p^2)$, resulting from the SD equation satisfied by the flavor-nonsinglet axial-vector vertex function and the corresponding WT identity[10].

As emphasized by Pagels[18] and Cornwall[16], corresponding to two concepts of mass, the mass operator $\Sigma(p^2)$ has two distinct contributions: $\Sigma_E(p^2,m)$ arises from the explicit breaking of chiral symmetry due to the current quark mass and $\Sigma_E = 0$ when $m = 0$; the dynamical mass operator $\Sigma_D(p^2)$ is independent of m and vanishes to any finite order in perturbation theory[41,43]. Since our main object is the NG realization in the symmetric theory, we shall set $m = 0$ from the beginning. In this case, $\Sigma = \Sigma_D$ satisfies the gap equation.

The $q\bar{q}$ Bethe-Salpeter scattering kernel $K(p,q,k)$ (described in terms of truncated four fermion diagrams which can not be disconnected by cutting two fermion lines), which occurs in the gap equation, contains two distinct pieces: $K = K_P + K_{NP}$, where K_P admits the familiar skeleton expansion and K_{NP} denotes the nonperturbative contributions arising from instantons. K_{NP} vanishes to any finite order in perturbation theory. It has been suggested[44] that K_{NP} is responsible for the NG realization of chiral symmetry in QCD. On the other hand, there also exist treatments of

the NG realization of chiral symmetry based on the ladder-approximated kernel wherein the instanton contributions are totally ignored[10,45].

The flavor nonsinglet, anomaly-free, renormalized, proper axial-vector vertex $\Gamma^{a}_{\mu 5}$ satisfies the WT identity

$$q^{\lambda}\Gamma^{a}_{\lambda 5}(p,p+q) = -\frac{Z_A}{Z_2}\left[\gamma_5 \frac{\lambda^a}{2} S^{-1}(p+q) + S^{-1}(p)\gamma_5 \frac{\lambda^a}{2}\right] \tag{3.3}$$

where we have defined the (unrenormalized) axial-vector vertex function by

$$(iS(p)\Gamma^{a}_{\lambda 5}(p,p+q)iS(p+q))_u = \frac{1}{3}\int d^4x\, d^4y\, e^{i[p.x-(p+q).y]}$$

$$\times \langle 0| T\left(\sum_{r=1}^{3} \Psi_r(x)\, J^{a}_{\lambda 5}(0)\, \bar{\Psi}_r(y)\right)|0\rangle , \tag{3.3a}$$

$$J^{a}_{\lambda 5} = \sum_{r=1}^{3} \bar{\Psi}_r \gamma_\lambda \gamma_5 \frac{1}{2}\lambda^a \Psi_r . \tag{3.3b}$$

Here the summation is over color indices, and λ^a is the flavor SU (N_F) matrix with $a = 1,2,\ldots,N_F^2 -1$. Constants Z_2 and Z_A renormalize (the unrenormalized) S^{-1} and $\Gamma^{a}_{\lambda 5}$ respectively by multiplying them. The inverse of the renormalized quark propagator has the form

$$S^{-1}(p) = A(p^2)\not{p} - \Sigma_D(p^2) \tag{3.4}$$

and possesses exact flavor symmetry:

$$[S^{-1}(p), \ \frac{\lambda^a}{2}] = 0, \ a = 1,2,\ldots,N_F^2 - 1 . \qquad (3.5)$$

Assuming that $SU_L(N_F)xSU_R(N_F)$ spontaneously breaks down to $SU_V(N_F)$, we have

$$\lim_{q\to 0} \quad q^\lambda \Gamma^a_{\lambda 5} (p,p+q) = - (\frac{Z_A}{Z_2}) \{ \gamma_5, S^{-1}(p) \} \frac{\lambda^a}{2}$$

$$\equiv (\frac{Z_A}{Z_2}) 2 \Sigma_D (p^2) \gamma_5 \ \frac{\lambda^a}{2}. \qquad (3.6)$$

This corresponds to a bound-state pseudoscalar pole at $q^2 = 0$ in $\Gamma^a_{\lambda 5}$, the bound state transforming according to the adjoint representation of $SU(N_F)$ and as a color singlet:

$$\Gamma^a_{\lambda 5} (p,p+q)|_{pole} \cong \frac{q_\lambda}{q^2} f_\pi \mathcal{P}(p,p+q) \gamma_5 \ \frac{\lambda^a}{2} . \qquad (3.7)$$

Here, $\mathcal{P}(p,p+q)\gamma_5$ is the quark-quark-pion vertex function, and f_π the pion decay constant. We can combine Equations (3.6) and (3.7) to get the Goldberger-Treiman relation,

$$f_\pi P(p^2) \equiv \lim_{q \to 0} \quad f_\pi \mathcal{P}(p,p+q) = (\frac{Z_A}{Z_2}) 2 \Sigma_D (p^2). \qquad (3.8)$$

The renormalized proper axial-vector vertex satisfies the SD equation

$$\Gamma^a_{\lambda 5} (p,p+q) = Z_A \gamma_\lambda \gamma_5 \ \frac{\lambda^a}{2} + i^2 \int(dk) \ [S(k) \Gamma^a_{\lambda 5}(k,k+q)$$

$$x \ S(k+q)] \ K(p,k,q) . \qquad (3.9)$$

When Equations (3.6) and (3.9) are combined together one gets the desired gap equation

$$\Sigma_D(p^2)\gamma_5 = -\int (dk)\left[S(k)\,\Sigma_D(k^2)\,\gamma_5\,S(k)\right] K(p,k,0). \tag{3.10}$$

For large momenta the perturbative contribution to the kernel is dominant and we can ignore the nonperturbative term. This feature of asymptotic freedom, emphasized by Lane[10], allows us to determine the large-momentum behavior of the complete kernel and consequently the large-momentum behavior of $\Sigma_D(p^2)$, provided $\Sigma_D(p^2)$ does not vanish identically. In the approximation of single-gluon exchange in the kernel, Equation (3.10) has two solutions (derived in Appendix F), which in the Landau gauge are

$$\Sigma_D(p^2) \underset{p^2\to\infty}{\sim} m_D \ln^{-\gamma}(p^2/\mu^2),\ \gamma = \frac{12}{33-2N_F} \quad \text{(irregular)} \tag{3.11a}$$

$$\sim \frac{4\,m_D^3}{p^2} \ln^{\gamma}(p^2/\mu^2) \quad \text{(regular)} \tag{3.11b}$$

The regular solution corresponds to typical bound-state behavior, while the irregular solution corresponds to an elementary point-like ground-state pseudoscalar meson. Brodsky and Farrar[46] have shown that simple power counting in the vector-gluon theory with the regular solution implies the pion electromagnetic form factor $F_\pi(q^2) \sim O(1/q^2)$ for asymptotically large q^2 ($F_H(t) \sim t^{1-n_H}$ for minimum number of fields n_H in the hadron). This is also the

experimentally observed behavior. For the irregular solution one gets $F_\pi(q^2) \sim O(1)$. Thus the usual experimentally observed bound-state physics of falling form factors, transverse momentum distributions, etc. can not be maintained with the irregular solution. So, following Pagels[43], we conclude that the experiment favors the regular solution. In (3.11b), m_D, which is a constant that sets the scale for DSB, will be the definition of the dynamical quark mass[47]. Using the asymptotic form $\Sigma_D(p^2) \sim 4m_D^3/p^2$, the amplitude below the strangeness threshold in e^-e^+ annihilation has been fitted with the conclusion that $m_D \simeq 244$ MeV. According to a better estimate, using the logarithmic part of (3.11b) as well[48], $m_D \simeq 315$ MeV.

As noted in the introduction of this chapter, one can also proceed directly via the SD equation for the quark propagator and find out a χSB solution in a confining QCD. By confining QCD we mean one in which the effective gluon propagator could be written as in QED but with the replacement (modulo gauge terms)

$$-g_{\mu\nu}\ \frac{e^2}{k^2} \rightarrow -g_{\mu\nu}\frac{\bar{g}^2(k^2)}{k^2}C_2$$

where $\bar{g}^2(k^2) \sim 1/k^2$ as $k^2 \rightarrow 0$ and $C_2 = \frac{4}{3}$ is the quark Casimir eigenvalue. This singular form, in a rough sense, corresponds to a linearly rising potential at large distances in configuration space. This form of the gluon propagator has been

advocated by several authors[34, 49-51]. In particular, Baker et al.[49] have found that in the infrared region in axial gauges singular part of the gluon propagator has the same spin structure as that of the free propagator and (the unrenormalized one in euclidean space) is given by

$$D^{(s)}_{\mu\nu}(q) = -\frac{Z(m)\, A M^2}{q^4}\left(\delta_{\mu\nu} - \frac{q_\mu n_\nu + q_\nu n_\mu}{q.n} + \frac{q_\mu q_\nu n^2}{(q.n)^2}\right) \qquad (3.12)$$

where Z(m) is the renormalization constant for the gluon wave function at the renormalization point M and A is some other constant. Ball and Zachariasen[35] (BZ) have used Equation (3.12) in the SD equation

$$S^{-1}(p) = S_0^{-1}(p) + g_0^2\, C_2 \int (dk)\gamma_\mu\, D_{\mu\nu}(k)\, S(p-k)\Gamma_\nu(p-k,p) \qquad (3.13)$$

satisfied by (unrenormalized) axial gauge quark propagator

$$S(p) = \not{p}\, F(p^2) + G(p^2) + (\not{p} H + I)\not{n} \qquad (3.14)$$

whereas the free propagator is $S_0(p) = \not{p}/p^2$. For $S(p-k)\Gamma_\nu(p-k,p)S(p)$, BZ write an expression which is its longitudinal part chosen in a way that is consistent with the WT identity (which, in axial gauge, has the same form as in Abelian gauge theories):

$$S(p')\Gamma_{\nu}^{(L)}(p',p)\,S(p) = -\Big[\Big(\frac{F+F'}{2}\Big)\gamma_{\nu} + \Big(\frac{F'-F}{2}\Big)\frac{2\not{p}'\gamma_{\nu}\not{p}+(p'^2+p^2)\gamma_{\nu}}{p'^2-p^2}$$

$$+\frac{G'-G}{p'^2-p^2}(\gamma_{\nu}\not{p}+\not{p}'\gamma_{\nu})\Big] + \not{n}\ \text{terms} \tag{3.15}$$

Here $p' = p-k$, and $F = F(p)$, $F' = F(p')$, etc. have been used. The equation thus obtained from Equation (3.13) can be broken into two equations which are equations for F and G separately if we choose the gauge p.n=0. We have taken this opportunity to work out the derivation of F and G (given in Appendix G), and found out that the correct expressions for the (renormalized) functions F and G are slightly different from the ones found by BZ:

$$F(p^2) = -\frac{1}{\beta M^2}\Psi(1;1;-p^2/\beta M^2) \tag{3.16a}$$

$$G(p^2) = C_1\,\Phi(2;\ 3/2;\ -p^2/\mu M^2) \tag{3.16b}$$

where $\beta \equiv AC_2Z(M)g_0^2/4\pi^2$. According to BZ, functions H and I may be dropped in case one is working in the gauge p.n=0. Henceforward, we shall assume this special choice of gauge for convenience, wherein we may set H = I = 0. To find out C_1, instead of following BZ, we shall follow the normalization procedure introduced by Cornwall[16] (remembering that our Green's functions are euclidean):

$$C_1^{-1} = S^{-1}(0) \simeq 300\ \text{MeV} \tag{3.17}$$

If we take the asymptotic behavior of (3.14) using (3.16) and compare it with the usual euclidean form of fermion propagator for asymptotically large momenta, then we find that $\bar{m}(p^2) \propto 1/p^2$ which may be compared to (3.11b). Thus we see that the quark propagator defined by Equations (3.14) and (3.16) has the correct momentum dependence even for asymptotically large momenta.

3.3 Dynamical Quarks Mass and Hadron Physics

As stated in the beginning of this chapter, nonperturbative dynamical quark mass plays important role in low energy hadron physics. This has been shown in quite a few works. Thus there are calculations of pion decay constant[16,47,53] and electromagnetic form factor of pion[47] More recently it has been used for the resolution of the U(1) problem in QCD and calculation of the $\pi^0 \to \gamma\gamma$ amplitude[53]. The results thus obtained are encouraging. We shall illustrate it by calculating the pion decay constant[15,16].

The axial-vector current for quarks is defined by Equation (3.3 b)

$$J^a_{\mu 5}(x) = \bar{\Psi}(x) \frac{1}{2} \lambda^a \gamma_\mu \gamma_5 \Psi(x), \qquad (3.3b)$$

with a trace over color indices understood. In the following we shall set renormalization constants $Z_A, Z_2 = 1$, and also $A(p^2) = 1$ in Equation (3.4). λ^a's are now isospin matrices.

Pion decay constant f_π appearing in Equation (3.7) is given by

$$\langle \pi^a(q) | J^b_{\mu 5}(0) | 0 \rangle = -i\, q_\mu f_\pi \delta^{ab} \tag{3.18}$$

while quark-quark-pion vertex $\mathcal{P}(p,p+q)\gamma_5$ is given by

$$\int d^4x\, e^{ipx} \langle \pi^a(q) | T(\Psi(x)\, \bar{\Psi}(0)\lambda^b) | 0 \rangle = iS(p)\mathcal{P}(p,p+q)\gamma_5\, iS(p+q)\delta^{ab} \tag{3.19}$$

Trace over color indices is understood. When p-integral of (3.19) is substituted in (3.18), one readily deduces

$$iq_\mu f_\pi \delta^{ab} = \int (dp)\, \mathrm{Tr}\left[iS(p) \frac{\lambda^a}{2}\mathcal{P}(p,p+q)\gamma_5\, iS(p+q) \frac{\lambda^b}{2}\gamma_\mu \gamma_5 \right] \tag{3.20}$$

where the trace is over Dirac, flavor and color indices, and on the right the limit is to be taken as $q \to 0$. Next, we expand r.h.s. of (3.20) retaining only linear terms in q. With the approximation that the regular part of axial-vector vertex is $\gamma_\mu \gamma_5 \lambda^a/2$, one has

$$f_\pi \frac{\partial}{\partial q^\nu} \mathcal{P}(k,k+q)\Big|_{q=0} \gamma_5 = -\gamma_5 \partial_\nu S^{-1}(p) - \gamma_\nu \gamma_5 \, . \tag{3.21}$$

When Equations (3.21) and (3.8) are substituted in r.h.s. of Equation (3.20) in linear terms in q, it takes the form

$$f_\pi^2 = -\,12i \int (dk)\, \frac{\Sigma_D(k^2)}{[k^2 - \Sigma_D^2(k^2)]^2} \left[\Sigma_D(k^2) - \frac{1}{2} k^2 \frac{d}{dk^2} \Sigma_D(k^2) \right]. \tag{3.22}$$

This last equation has been used in References [16,47], and in a slightly modified form in Reference [53] to evaluate f_π. In Reference [47], $\Sigma_D(p^2) \approx \frac{4m_D^3}{p^2}$ with $m_D \simeq 244$ MeV has been used with the result that $f_\pi = 83$ MeV, close to the experimental value of 93 MeV. Similar results have been obtained in the other two references using slightly modified forms of Σ_D to take into account its infrared behavior.

Chapter 4

ANOMALOUS MAGNETIC MOMENTS OF LIGHT QUARKS AND DYNAMICAL SYMMETRY BREAKING

4.1 Introduction

The magnetic moments of the proton and neutron are usually calculated in terms of confined constituent-quarks. A better knowledge of the magnetic moments of these baryons requires a better knowledge of the anomalous magnetic moments (a.m.m.'s) of their constituent quarks. As noted in Chapter 3, dynamically generated quark mass plays an important role in low energy hadron physics. In view of this fact, it will be interesting to study the effect of the introduction of dynamical quark mass on the a.m.m.'s of light quarks u and d, which we can assume, to a good approximation, to have vanishing current quark masses. The content of this chapter has been taken from Reference [54].

In QED, the a.m.m. term (in the lowest order) has the form $\frac{i}{2m} \sigma_{\mu\nu} q^{\nu} \frac{\alpha}{2\pi}$. In QCD where chiral symmetry has been broken dynamically, one naively expects the same form except for the change $m \to m_{dyn}$ and $\alpha \to \alpha_s C_2$. But the dynamically generated mass depends upon the coupling constant in a nonanalytic manner such that it vanishes in the limit of vanishing coupling[10,11]. That is, even for small couplings, we expect a.m.m. term in such a theory to be

pretty large. We shall illustrate this by an example.

Consider a modified form of Nambu-Jona-Lasinio model given by the Lagrangian

$$\mathcal{L} = \bar{\Psi}\,(i\not{\partial} - e\not{A})\Psi - 1/4(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + g[(\bar{\Psi}\Psi)^2 + (\bar{\Psi}\, i\gamma_5 \Psi)^2] \tag{4.1}$$

which is invariant under ordinary local phase transformation

$$\Psi(x) \to \exp(i\theta(x))\,\Psi(x), \quad A_\mu \to A_\mu - \partial_\mu\,\theta(x)/e$$

as well as chiral (global) transformation

$$\Psi(x) \to e^{i\gamma_5\theta}\,\Psi(x), \quad \bar{\Psi}(x) \to \bar{\Psi}(x)\, e^{i\gamma_5\theta} .$$

Gauge terms will be considered as a small perturbation over the original field theory. After the chiral symmetry has been broken dynamically, the fermion develops a mass $\Sigma\,(p) = m$ given by[9] (neglecting the effect of the gauge interaction)

$$\frac{2\,\pi^2}{g\,\Lambda^2} = 1 - \frac{m^2}{\Lambda^2}\ln\left(\frac{\Lambda^2}{m^2} + 1\right) \tag{4.2}$$

where Λ is an ultraviolet cut-off and $0 < \frac{2\pi^2}{g\Lambda^2} < 1$ for getting real Λ/m. Next, we shall use the resultant fermion propagator

$$S(p) = \frac{1}{\not{p} - \Sigma\,(p)} \tag{4.3}$$

to calculate the lowest order vertex diagram given in Figure 1. This gives the a.m.m. term:

$$\Gamma_\mu(p,p+q) - \gamma_\mu = 2ig\int(dk) \frac{i}{\not{k} - \Sigma(k)} \gamma_\mu \frac{i}{\not{k} + \not{q} - \Sigma(k+q)}$$

$$= \frac{i\,\sigma_{\mu\nu}\, q^\nu}{2m} \frac{1/2}{\frac{2\pi^2}{g\Lambda^2}} \left[\frac{2\pi^2}{g\Lambda^2} - \frac{\Lambda^2}{\Lambda^2 + m^2} \right]. \tag{4.4}$$

If $2\pi^2/g\Lambda^2$ is chosen very small then a.m.m. indeed becomes very large.

In the literature various forms of 'running' (dynamical) quark masses have been derived and used for calculating low-energy hadronic quantities. We shall use them in the quark propagator in the following SD equation satisfied by the quark-electromagnetic-current vertex function

$$\Gamma_\mu(p,p+q) = Z_2\gamma_\mu - \int(dk)\, [S(k)\Gamma_\mu(k,k+q)S(k+q)]\, K(p,q,k)\,. \tag{4.5}$$

QED will also be assumed to be operative but its effect on the mass splitting of u and d quarks, and its contributions to K will be ignored.

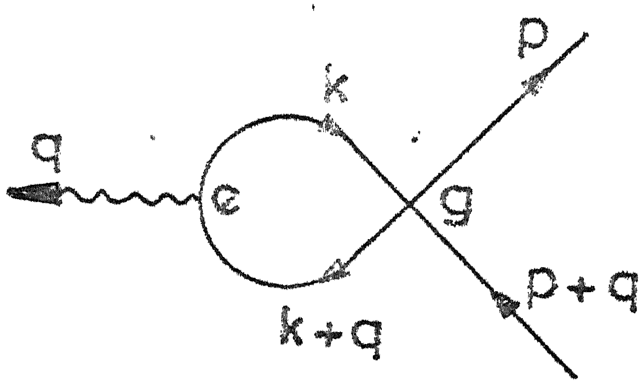

Fig. 1

4.2 Solutions Based on Asymptotic Properties of QCD

In this section we shall first use the asymptotic solution (3.11b) (with $\Sigma_D \rightarrow \Sigma$ and $m_D \rightarrow m$) ignoring the logarithmic factor. Also $A(p^2)$ in Equation (3.4) will be taken to be unity. This kind of solution has been used by Pagels and Stokar[47] to calculate the pion decay constant (as shown in the last part of Chapter 3) and the electromagnetic form factor of the pion successfully, and also to calculate the quark electromagnetic self-energy. A consistent way to use this solution for our purpose would be to calculate the lowest order vertex diagram given in Figure 2. When this is done with the replacement $\Sigma \rightarrow m$ in the denominator, it turns out that the Pauli form factor $F_2(0)$ defined by

$$\Gamma_\mu(p+q,p) = F_1(q^2)\gamma_\mu + \frac{i}{2m}\sigma_{\mu\nu} q^\nu F_2(q^2) \qquad (4.6)$$

becomes complex. Evidently the asymptotic form of Σ, which we have used, mimicks a massless scalar propagator and is not suitable in the low energy region of integration. As a remedy, following Cornwall[16], we shall parametrize $\Sigma(p^2)$ as

$$\Sigma(p^2) = \frac{M\Lambda^2}{\Lambda^2 - p^2} . \qquad (4.7)$$

This gives the following expression:

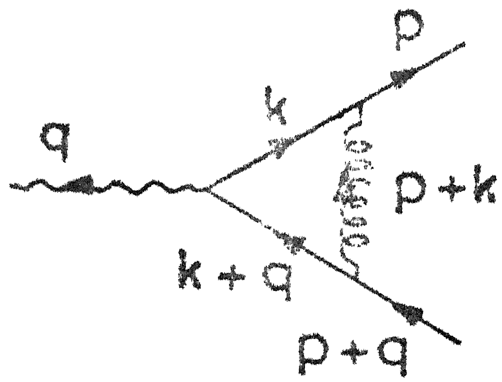

Fig. 2

$$F_2(q^2) = - \frac{g^2\ C_2\ m}{4\ \pi^2} \int_0^1 dx \int_0^1 dy \left[\frac{-m(1-x)}{m^2-q^2y(1-y)} \right.$$

$$\left. + \frac{2m\ (1-x)}{m^2x + (\Lambda^2 - m^2)(1-y)-q^2xy(1-y)} \right] \quad (4.8)$$

which, for $q^2=0$, reduces to

$$F_2(0) = \frac{g^2C_2}{4\pi} \frac{1}{2\pi} + \frac{g^2C_2}{4\pi} \frac{m^2}{\pi(\Lambda^2 - m^2)^2} x$$

$$\left[\ln \frac{m^2}{\Lambda^2-m^2} - \frac{\Lambda^4}{m^4} \ln \frac{\Lambda^2}{\Lambda^2-m^2} + \frac{\Lambda^2}{m^2} - 1 \right] \quad (4.9a)$$

$$\equiv \frac{g^2C_2}{4\pi^2} f\ (\Lambda^2/m^2) \quad (4.9b)$$

where $m = \Sigma(p^2)\big|_{\not p=m} = \frac{M\Lambda^2}{\Lambda^2-m^2}$, provided that it can be solved for m for a given M and Λ.

The first term on r.h.s. of (4.9a) is the standard result whereas the second term becomes complex for $m > \Lambda$.

The plot of f vs. Λ^2/m^2 (Figure 3) shows that it starts with a negative value and at $\Lambda^2/m^2 \simeq 7.8$, it becomes zero. As Λ^2/m^2 becomes infinitely large, it tends to 1/2, the standard result. Cornwall[16] has given an estimate of $M \simeq 300$ MeV and $\Lambda \simeq 600$ MeV. But with this set of values, a solution to the equation $m = \frac{M\Lambda^2}{\Lambda^2-m^2}$ does not exist. As an example, the following set of values satisfy this equation: $M \simeq 300$ MeV, $\Lambda \simeq 900$ MeV and $m \simeq 360$ MeV;

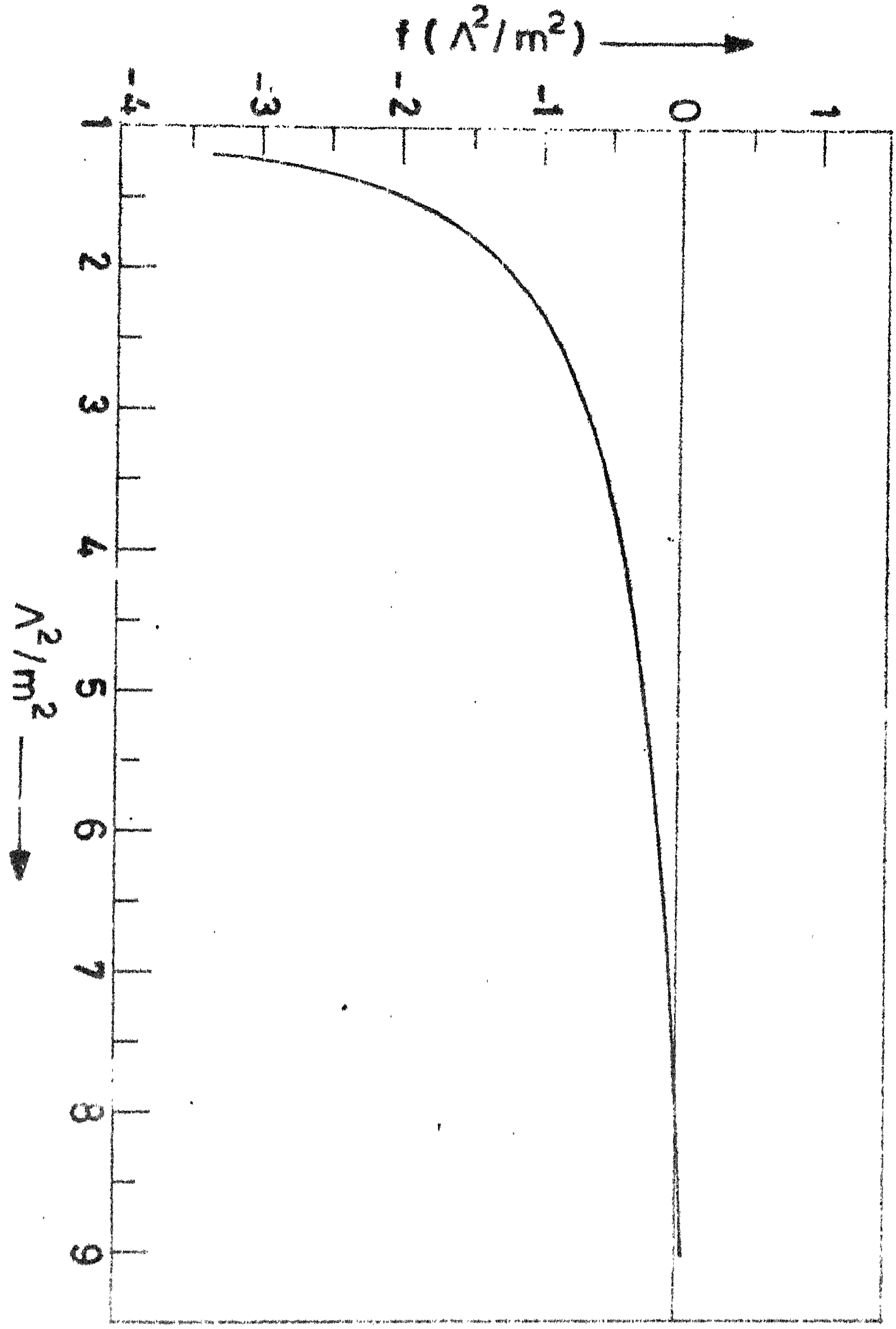

Fig. 3

and for this set $f(\Lambda^2/m^2) \simeq -0.11$. For the accepted value[55] α_s (1 GeV^2) ≈ 0.50 (and taking maximum mass scale Λ above as the argument of α_s in (4.9b)), one finds $F_2(0) \approx -1/45$.

4.3 Solutions Based on Infrared Properties of QCD

In this section we shall use a singular effective form of the gluon propagator. For the XSB quark propagator, we shall use either a parametric form given by Equation (4.7) or a form consistently derived using such a singular gluon propagator. Furthermore, following Acharya and Narayana-Swamy[36], we shall assume for small momenta, where K_{NP} is expected to dominate over K_P that the leading (most singular) term in K has the ladder form with a singular effective gluon propagator, $D_{\mu\nu}(k) \sim k^{-4}$. "This approximation to the dynamics may be regarded as an effective strong coupling approximation". Works of Cornwall[16] and Richardson[51] support this hypothesis.

The form of gluon propagator introduced by Baker et al.[49] (Equation (3.12)) and the resultant XSB quark propagator derived by Ball and Zachariasen[35] (BZ) (corrected form Equations (3.16)) is the one which we shall use for this calculation. Again we shall evaluate the one-gluon-exchange graph of Figure 2, but here the quark-photon vertex will be the complete one (all the calculations below are in euclidean metric):

$$\Gamma_\mu(p,p+q) = Z_2\gamma_\mu + g_0^2 C_2 \int (dk)\gamma_\lambda S(k)\Gamma_\mu(k,k+q)S(k+q)\gamma_\sigma D_{\lambda\sigma}(p-k). \tag{4.10}$$

Here we shall approximate the full quark-photon vertex by its longitudinal part. Since the quark-photon and the quark-gluon vertices obey similar WT identities, following Reference [36], identical longitudinal decomposition can be written for both the vertices (except for the difference that the quark-gluon vertex will have a color matrix), at least for small momentum transfer. Thus, we shall write[35]

$$S(k)\Gamma_\mu^{(L)}(k,k')\,S(k') = -\Big[\tfrac{1}{2}(F+F')\gamma_\mu + \tfrac{1}{2}(F'-F)\times \frac{2\,\not{k}\gamma_\mu\not{k}' + (k^2+k'^2)\gamma_\mu}{k'^2-k^2} + (G'-G)\frac{\gamma_\mu\not{k}' + \not{k}\gamma_\mu}{k'^2-k^2}\Big] \tag{4.11}$$

for the quark-photon vertex as well for small $q = k'-k$. It is to be noted that if the mass-shell condition is not used (which is the case for the confined quarks we are dealing with), then the contribution to the Pauli form factor can come only from the last term in the square bracket of Equation (4.11). Thus we are interested in calculating the following integral

$$I_\mu = \int \frac{d^4k}{(k-p)^4} \quad \frac{G(k'^2) - G(k^2)}{k'^2 - k^2} \gamma_\lambda (\gamma_\mu \not{k}' + \not{k}\gamma_\mu) \gamma_\sigma \times$$

$$[\delta_{\lambda\sigma} - \frac{(k-p)_\lambda n_\sigma + (k-p)_\sigma n_\lambda}{(k-p).n} + \frac{(k-p)_\lambda (k-p)_\sigma n^2}{((k-p).n)^2}] . \tag{4.12}$$

Details of the angular integration have been worked out in Appendix H. Choosing all euclidean γ-matrices antihermitian, the result of the integration is

$$I_\mu = q_\nu [\gamma_\nu, \gamma_\mu] \pi^2 \{(1 + \frac{p.q}{q^2})(2 - \frac{p.q+q^2}{(p+q)^2}) \int_{p^2}^{(p+q)^2} dk^2 \frac{G(k^2)}{(k^2+p^2+p.q)^2}$$

$$- \int_0^{q^2/4} dk^2 \; G(k^2) [\frac{(q^4 - 4k^2q^2)^{1/2}}{q^2(k^2+p^2+p.q)^2} + \frac{2}{(q^4-4k^2q^2)^{1/2}(k^2+p^2+p.q)}]\}$$

$$+ q_\mu \text{ terms} + n_\mu \text{ terms} \tag{4.13}$$

where the constant vector n has been chosen to be perpendicular to the hyperplane defined by p,q and $(i\sigma_{\mu\nu} q_\nu)$. We have not been able to locate any formula in the literature which can be used to perform various integrations, which appear in (4.13), analytically, if the transcendental form given by Equation (3.16b) is used for $G(k^2)$. However, the integration can be done in certain limiting cases:

(i) When $p^2, q^2 << \beta M^2$ so that the series expansion for Φ function is valid and it can be approximated by the first few terms in the series; then

$$I_\mu = q_\nu[\gamma_\nu,\gamma_\mu]C_1 \;\pi^2\{(1+\frac{p.q}{q^2})(2-\frac{p.q+q^2}{(p+q)^2})[(1+\frac{4}{3}\frac{p^2+p.q}{\beta M^2}$$

$$+\frac{4}{5}\frac{(p^2+p.q)^2}{\beta^2 M^4})(\frac{1}{p.(2p+q)}-\frac{1}{(p+q).(2p+q)})+\frac{4(q^2+2p.q)}{5\beta^2 M^4}$$

$$-(\frac{4}{3\beta M^2}+\frac{8(p^2+p.q)}{5\beta^2 M^4})\ln\frac{(p+q).(2p+q)}{p.(2p+q)}]-\frac{4}{q^2 r}$$

$$-\frac{4}{3\beta M^2}[2-(r+1)^{1/2}\ln\frac{(r+1)^{1/2}+1}{(r+1)^{1/2}-1}]$$

$$-\frac{1}{5}\frac{q^2}{\beta^2 M^4}[\frac{4}{3}+4r-2r(r+1)^{1/2}\ln\frac{(r+1)^{1/2}+1}{(r+1)^{1/2}-1}]\}$$

$$+q_\mu \text{ terms} + n_\mu \text{ terms} \qquad (4.14a)$$

where $r = \frac{4p.(p+q)}{q^2}$. Now on taking the $p \to 0$ limit:

$$I_\mu \to -q_\nu[\gamma_\nu,\gamma_\mu]C_1\frac{4\pi^2}{3\beta M^2}(1-\frac{2q^2}{5\beta M^2}) + q_\mu\text{terms} + n_\mu\text{terms} \qquad (4.14b)$$

$$\xrightarrow[q^2\to 0]{} -q_\nu[\gamma_\nu,\gamma_\mu]C_1\frac{4\pi^2}{3\beta M^2} + q_\mu\text{terms} + n_\mu\text{terms}. \qquad (4.14c)$$

This gives us

$$\Gamma_\mu(0,q) = -i\sigma_{\mu\nu}q_\nu\frac{C_1}{2}\frac{4}{3} + \gamma_\mu \text{ terms} + \ldots \qquad (4.15)$$

where dots indicate other possible Lorentz structures.

(ii) When $q \to 0$ (but with finite p):

$$I_\mu = -q_\nu[\gamma_\nu,\gamma_\mu]_- \frac{\pi^2}{p^2} \left\{\frac{G(0)}{2} - G(p^2)\right\} + q_\mu \text{terms} + n_\mu \text{terms} \tag{4.16}$$

This gives,

$$\Gamma_\mu(p,p+q) = -i\,\sigma_{\mu\nu} q_\nu \frac{\beta(M^2)M^2}{2p^2} C_1\left[1/2 - \Phi(2;3/2;-\frac{p^2}{\beta M^2})\right]$$

$$+\gamma_\mu \text{terms} + \ldots\ldots \tag{4.17a}$$

If we substitute numerical values for the quantities appearing in Equation (4.17a), namely, $\beta(M^2)M^2 \simeq \frac{8}{3\pi} \times (0.16\ \text{GeV})^2$ [56] and $p^2 \simeq (0.3\ \text{GeV})^2$ [16,55], then we get

$$\Gamma_\mu(p,p+q) \simeq - \frac{i\,\sigma_{\mu\nu} q_\nu}{2\times 300\ \text{MeV}} \times \frac{1}{9} + \gamma_\mu \text{ terms} + \ldots\ldots \tag{4.17b}$$

Similarly,

$$\Gamma_\mu(p+q,p) \simeq \frac{i\,\sigma_{\mu\nu} q_\nu}{2\times 300\ \text{MeV}} \times \frac{1}{9} + \gamma_\mu \text{ terms} + \ldots\ldots \tag{4.17c}$$

In minkowskian metric, this can be written as

$$\Gamma_\mu(p+q,p)\Big|_{\text{mink}} \simeq - \frac{i\,\sigma_{\mu\nu} q^\nu}{2\times 300\ \text{MeV}} \times \frac{1}{9} + \gamma_\mu \text{ terms} + \ldots\ldots \tag{4.17d}$$

where we have followed the common practice[55] in assuming the same form for $\bar{m}_{dyn}(p^2)$ in the spacelike

region ($p^2 < 0$) as in the timelike region.

We have taken the same form (Equation (4.11)) for the longitudinal part of the both quark-gluon and quark-photon vertices. Assuming that their transverse parts are not much different from each other, we can write the counterpart of Equation (4.17d) for the quark-gluon case as

$$\Gamma_\mu(p+q,p) \simeq - \frac{i\sigma_{\mu\nu}q^\nu}{2\text{x}300\ \text{MeV}} \text{ x } \frac{C}{9} + \gamma_\mu \text{ terms } + \ldots \qquad (4.17e)$$

where C is a number, likely to be O(1), which takes into account the non-Abelian character of the gluon. There will be a similar change in Equation (4.15).

(iii) When $q^2 >> p^2 \sim \beta M^2$: Since momentum transfer is no longer small, strictly speaking relation (4.11) can be applied only for the quark-gluon vertex. We are assuming that when Equations (4.11), (4.12) and (4.13) are used in Equation (4.10), the result so obtained can be extrapolated to large q even for the quark-photon vertex . Thus we shall approximate the quark-photon vertex by

$$\Gamma_\mu(p,p+q) \simeq -i\sigma_{\mu\nu}q_\nu \frac{\beta^2(M^2)M^4C_1}{4p^2q^2} + \gamma_\mu \text{terms} + \ldots \qquad (4.18a)$$

$$\Gamma_\mu(p,p+q)\Big|_{\text{mink}} \simeq -i\,\sigma_{\mu\nu}q^\nu \frac{\beta^2M^4C_1}{4p^2q^2} + \gamma_\mu \text{ terms } + \ldots \qquad (4.18b)$$

$$\Gamma_\mu(p+q,p) \simeq i\sigma_{\mu\nu} q_\nu \frac{\beta^2 M^4 C_1}{4p^2 q^2} + \gamma_\mu \text{ terms } + \ldots \quad (4.18c)$$

$$\Gamma_\mu(p+q,p)\Big|_{mink} \simeq i\sigma_{\mu\nu} q^\nu \frac{\beta^2 M^4 C_1}{4p^2 q^2} + \gamma_\mu \text{ terms} + \ldots\, . \quad (4.18d)$$

Similarly, we assume without explicitly calculating the triple gluon vertex contribution, that the quark-gluon vertex has the same form [as Equations (4.18a)-(4.18d)] apart from a constant factor, C', O(1).

If we use expression (4.7) for $\Sigma(p^2)$ (and the mass-shell condition) along with the singular part of gluon propagator $D_{\mu\nu}(k) \sim g_{\mu\nu} \frac{m'^2}{k^4} \rightarrow g_{\mu\nu} \frac{m'^2}{(k^2-\mu^2+i\varepsilon)^2}$, which has been used by Cornwall[16] for calculating f_π etc., then the Pauli form factor becomes linearly divergent as $\mu \rightarrow 0$. Cornwall has argued that μ should be kept finite and equal to a typical hadronic mass scale. On the other hand if, following Cornwall, quarks are assumed to be confined particles having no mass shell, then the Pauli term does not arise.

Acharya and Narayana-Swamy[36] have also examined the feasibility of dynamically broken chiral symmetry in QCD with zero-bare-mass quarks interacting via single gluon exchange when the gluon propagator has the infrared behavior: $D_{\mu\nu}(k) \sim k^{-4}$. They have found that the chiral symmetry can be, although not necessarily, realized in the Nambu-Goldstone mode with

$$S^{-1}(p) = -m(p^2) = \text{constant} . \tag{4.19}$$

As can be easily checked, this result is too strong to give the Pauli form factor.

4.4 Experimental Consequences of Anomalous Magnetic Moment Term

In this section we shall explore some of the experimental consequences of the presence of Pauli terms given by Equations (4.17) and (4.18).

(a) Baryon magnetic moment: If we make the usual assumption that the baryon moments arise solely from the constituent-quark moments, then following Barik and Das[57], and references given therein, we can obtain expressions for the magnetic moments of the proton and neutron in terms of the magnetic moments of the corresponding constituent quarks. This has been done by writing the baryon magnetic moment as

$$\mu_B = \sum_q \langle B\uparrow | \; \mu_q \; \sigma_z^q \; | B\uparrow \rangle \tag{4.20}$$

where $|B\uparrow\rangle$ stands for the state vectors of the baryon in question and in the present case, it represents the regular SU(6) state vectors. The well known relations between the baryon magnetic moments and the corresponding constituent-quark

moments are:

$$\mu_p = \frac{1}{3}(4\mu_u - \mu_d), \ \mu_n = \frac{1}{3}(4\mu_d - \mu_u). \qquad (4.21)$$

If we take a simple expression for the quark (Dirac) magnetic moment in units of the nuclear magneton as

$$\mu_q^D = \frac{M_p}{m_q} e_q \qquad (4.22)$$

where M_p is the proton mass, m_q, the effective mass of the quark defined by Equation (3.17), and e_q, the electric charge of the quark in the unit of the proton charge. Thus from Equations (4.22) and (4.17d) we have (in nuclear magnetons)

$$\mu_q = \frac{M_p}{m_q} e_q (1 - 1/9) \qquad (4.23a)$$

$$\mu_u \simeq 1.86, \quad \mu_d \simeq - 0.93 . \qquad (4.23b)$$

This result, when substituted in Equation (4.21), gives

$$\mu_p \simeq 2.79 \ , \ \mu_n \simeq -1.86 \qquad (4.24)$$

which can be compared with the experimental numbers 2.79 and -1.91 respectively. The agreement may be fortuitous, particularly in view of the fact that the value chosen for m_q was a bit uncertain,

but what is remarkable is the fact that the dynamical mass, given by Equation (3.17)(which was so chosen in a different context, namely, in the calculation of the pion decay constant[16]), together with a.m.m. given by (4.17d) (where again the same dynamical mass was used) can yield a number which is so close to the experimental one.

(b) Spin-dependent potential energy between quarks: Here we shall calculate the spin-dependent part of the potential energy between a heavy quark and a light antiquark. The light antiquark may be regarded as moving within the color field provided by the heavy quark. Our argument below will not be as airtight as we might wish. For the sake of simplicity, we shall depart from our earlier convention and assume that the quarks may be treated as on-mass-shell within the framework of a confining potential.*

---

*Arbuzhov[58] has shown that quarks may have a mass-shell even though the gluon propagator of $1/k^4$-type is used. Although our calculation of the Pauli term does not take this possibility into account, we shall do it here and in the following due to our igmorance of methodology in treating such cases otherwise.

A simple calculation of the spin-dependent part of the potential with one-gluon exchange in the non-relativistic limit shows[59] that

$$\tilde{V}_s(q) = -\frac{4}{3} Z(M) \frac{AM^2}{\vec{q}^{\,4}} g_o^2 \Big[i \frac{\vec{\sigma}_1 \cdot \vec{p}_1 x \vec{p}_1'}{4m_1^2} (1+2K_1)$$

$$+ \frac{i\vec{\sigma}_1 \cdot \vec{p}_1 x \vec{p}_1'}{2m_1 m_2} (1+K_1) + \frac{i\vec{\sigma}_2 \cdot \vec{p}_1 x \vec{p}_1'}{4m_2^2} (1+2K_2)$$

$$+ \frac{i\vec{\sigma}_2 \cdot \vec{p}_1 x \vec{p}_1'}{2m_1 m_2} (1+K_2)$$

$$- \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}) - \vec{q}^{\,2}\, \vec{\sigma}_1 \cdot \vec{\sigma}_2}{4m_1 m_2} (1+K_1)(1+K_2) \Big] \qquad (4.25a)$$

where $\vec{\sigma}_1$ and $\vec{\sigma}_2$ are Pauli matrices which act on particle 1 and 2 spin wave functions respectively; K_1 and K_2 are the (constant) chromomagnetic Pauli form factors and q is the momentum transfer: $q = p_1' - p_1$. Following the conventional wisdom that $1/q^4$-type of potential corresponds to a linearly rising potential in the configuration space[49-51], we can Fourier-transform Equation (4.25a) as

b- and c- quarks have large current-quark masses and hence K_b and K_c are, presumably, small; hence (4.26c) is essentially the result obtained by Eichten and Feinberg[60]. This result agrees well with the experimental numbers. However, Equation (4.25b) has fewer free parameters than Equation (6.2) of Eichten and Feinberg.

(c) Reaction cross-section of $p\bar{p} \to$ hadrons: Here, we shall calculate essentially the reaction cross-section of $q\bar{q} \to q\bar{q}$ through one-gluon exchange (Figures 4a and 4b). For quark-gluon vertices we shall use Equations (4.18), along with the constant factor C', and see the effect of the Pauli term over and above the bare vertex. Since here we are looking at the processes involving large momentum transfer, we shall use the conventional form of the gluon propagator $D_{\mu\nu}(q) \sim q^{-2}$. External quark and antiquark lines are assumed to be on mass-shell. If θ be the scattering angle and $(p+k)^2 = (2E)^2$, then the differential cross-section for this process in the center of mass frame (retaining only linear terms in Pauli form factor) is given by

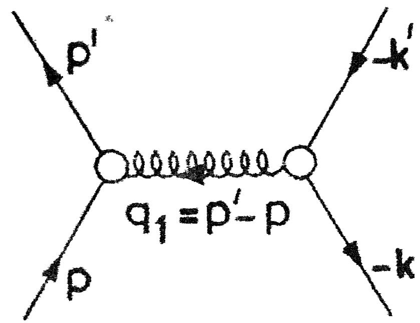

Fig. 4a

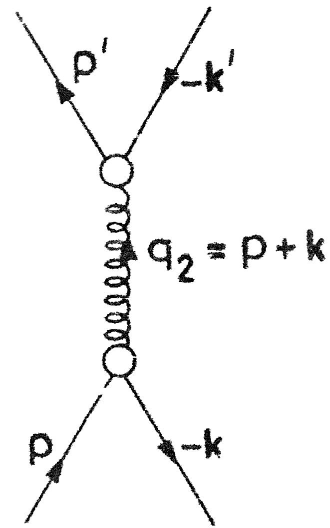

Fig. 4b

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{g^4\ 2/9}{128\pi^2 E^2}\left[\frac{1+\cos^4\theta/2}{\sin^4\theta/2} - \frac{2\cos^4\theta/2}{\sin^2\theta/2} + \frac{1}{2}(1+\cos^2\theta)\right] + \frac{g^4\ 2/9}{128\pi^2 E^2}\ \frac{\beta^2(M^2)M^4C_1}{mE^2} \times$$

$$\left[-\frac{1+\cos^4\theta/2}{2\sin^6\theta/2} + \frac{\cos^4\theta/2}{2\sin^4\theta/2} - \frac{\cos^4\theta/2}{2\sin^2\theta/2} + \frac{1}{2}\right]$$

$$+ \frac{g^4\ 2/9}{128\pi^2 E^2}\ \frac{C'\beta^2(M^2)M^4C_1}{mE^2}\left[\frac{1+\cos\theta}{2\sin^6\theta/2} + \frac{6+\sin^2\theta+2\cos\theta}{16\sin^4\theta/2} + \frac{\sin^2\theta+2\cos\theta}{8\sin^2\theta/2}\right] \tag{4.27}$$

where the first two terms are the contributions coming from the bare vertex part and the third term is the contribution from the Pauli term. It is clear that for high energies, the contribution of the third term can be significant only for very small values of θ. To get an estimate of the angle at which the contribution of the third term is of the same order or one order of magnitude smaller than that of the first term, we have

$$\frac{C'\beta^2 M^4 C_1}{mE^2}\ \frac{1}{2\sin^2\theta/2} \sim 1 \text{ or } 10^{-1}. \tag{4.28a}$$

Even for $m \simeq 10$ MeV (a number which is usually assigned at high energies) and $E \simeq 50$ GeV, this gives

$$\theta \sim \sqrt{10} \times 10^{-3} \text{ or } 10^{-2}. \tag{4.28b}$$

Even if we use the full Dirac form factor (instead of just unity), the crude estimate (4.28b) is unlikely to change. Thus it will be difficult to detect the effect of the Pauli term this way.

If, instead, we had looked at the differential cross-section for $e^- e^+ \to q\bar{q}$, then in the equation corresponding to Equation (4.28a), there would not have been any $(\text{Sin}\theta/2)^{-2}$ factor on l.h.s. and the effect of the Pauli term would have been insignificant at all angles.

4.5 Summary and Conclusion

We have seen that anomalous magnetic moments (of the fermions)can be significant, although not appreciably large as speculated in the beginning of this chapter (at least for the realistic cases), in theories where mass generation occurs through dynamical symmetry breaking. To calculate a.m.m.'s of light quarks we essentially used one-gluon-exchange approximation and employed solutions of both kinds: solutions based on asymptotic and infrared properties of QCD derived and used by various authors. We found that

all solutions can not be used for this calculation. Solution for the gluon propagator derived by Baker et.al.[49] and the resultant chiral symmetry breaking and confining solution for the quark propagator derived by Ball and Zachariasen[35] form a good combination for this calculation. The result thus arrived at was used to check the experimental consequences of the presence of a.m.m. (both electromagnetic as well as chromomagnetic) term. It gave very good results for proton and neutron magnetic moments when regular SU(6) state vectors for baryons were used. In fact the use of Equation (4.21) alone gives the correct ratio of magnetic moments of proton and neutron, using dynamical mass (here the effective quark mass) given by Equation (3.17) and the resulting a.m.m. given by Equation (4.17d), one obtains correct values for the individual baryons. In the case of the spin-dependent potential calculation, we obtained a result which agreed with an earlier calculation (which had a good agreement with experimental results) and, moreover, had fewer parameters. In differential cross-section calculation of $q\bar{q} \to q\bar{q}$, we found that the effect of the presence of the Pauli term was insignificant (at high energy) at any angle $\theta \gtrsim 10^{-2}$, while in case of $e^-e^+ \to q\bar{q}$ the effect of the Pauli term was insignificant everywhere.

Although we have worked in the axial gauge (with a specially chosen direction of the axial vector n), our expression for a.m.m. is apparently free of n .

Chapter 5

UP- AND DOWN- QUARK MASS DIFFERENCE AND DYNAMICAL SYMMETRY BREAKING

Encouraged by the applications of the dynamical quark mass as illustrated in Chapter 3 and the results derived in Chapter 4, we shall use the dynamical quark mass to calculate u-d quark mass difference. This is the modern version of the old problem of p-n mass difference. If the origin of the quark mass lies in the dynamical breaking of the chiral symmetry of the QCD Lagrangian, then this may provide a mechanism through which QCD plays a role in determination of u-d quark mass difference (taken to be of electromagnetic origin) through its feed-back on electromagnetic self-energy. With a new interaction coming into picture, one may hope to get the correct sign of u-d quark mass difference.

Let us substitute Equation (3.4) with $A(p^2) = 1$, in the gap Equation (3.10). Then one gets (dropping the subscript D in Σ_D):

$$\gamma_5 \Sigma(p^2) = \int (dk) \left[\frac{\gamma_5 \Sigma(k^2)}{k^2 - \Sigma^2(k^2)} \right] K(p,k,0) \, . \tag{5.1}$$

With K as the QCD kernel for $q\bar{q}$ scattering, for large momenta in the approximation of single-gluon-exchange, the solution of Equation (5.1) was found to be (3.11b).

To proceed further, we follow Reference [61].

Consider the self-energy of a quark obeying the integral Equation (5.1) where K includes both QCD and QED interactions. We assume the QED interaction to be a perturbation to the QCD interaction, i.e., that $K = K_o + \delta K$, where K_o is the QCD kernel in the lowest order and $\delta K = \delta K_{EM} + \delta K_{\delta\Sigma}$. Here δK_{EM} is explicitly the QED kernel for $q\bar{q}$ scattering. $\delta K_{\delta\Sigma}$ is the effect on the kernel due to $\delta\Sigma$, the change in the mass operator. Again we shall approximate δK by the lowest-order term in δK_{EM}. The lowest-order term in $\delta K_{\delta\Sigma}$ has been neglected because we have already neglected the two-gluon-exchange contribution to K_o. Writing $\Sigma = \Sigma_o + \delta\Sigma$ we get the following integral equation for $\delta\Sigma$:

$$\gamma_5 \, \delta\Sigma \; (p^2) = \int (dk) [\; \gamma_5 \delta\Sigma \; (k^2) \; \frac{k^2 + \Sigma_o^2 \; (k^2)}{[k^2 - \Sigma_o^2 \; (k^2)]^2} \;] K_o (p,k,0)$$

$$+ \int (dk) [\frac{\gamma_5 \, \Sigma_o \; (k^2)}{k^2 - \Sigma_o^2 (k^2)}] \; \delta K_{EM} \; (p,k,0) \; . \qquad (5.2)$$

The second term which is the driving term in the above equation is exclusively $\delta\Sigma_{EM}$. After performing a Wick rotation and doing the angular integration, Equation (5.2) can be written (in the Landau gauge) as

$$\delta\Sigma_i\ (p^2) = \frac{3\ C_2}{16\pi^2} \int_0^{p^2} \frac{g_s^2(k^2)}{p^2} \frac{[\,k^2 - \Sigma_o^2(k^2)]}{[\,k^2 + \Sigma_o^2(k^2)\,]^2}\ \delta\Sigma_i(k^2)k^2\ dk^2$$

$$+ \frac{3\ C_2}{16\pi^2} \int_{p^2}^{\infty} g_s^2(k^2)\ \frac{[k^2 - \Sigma_o^2\ (k^2)]}{[k^2 + \Sigma_o^2(k^2)\,]^2}\ \delta\Sigma_i(k^2)dk^2$$

$$+ \frac{3\ e_i^2}{16\pi^2} \int_o^{p^2} \frac{\Sigma_o^2\ (k^2)\ k^2\ dk^2}{p^2\,[k^2 + \Sigma_o^2(k^2)\,]} + \frac{3\ e_i^2}{16\pi^2}\ x$$

$$\int_{p^2}^{\infty} \frac{\Sigma_o\ (k^2)\ dk^2}{k^2 + \Sigma_o^2\ (k^2)} \tag{5.3}$$

where we have let $g_s^2((k-p)^2) \rightarrow g_s^2(k^2)$ and $i=(u,d)$. For large p^2, Equation (5.3) can be converted into a differential equation (upto terms of order $\Sigma_o^2(p^2)/p^2$ which we will ignore) of the form:

$$[p^2\ \delta\Sigma\, i\ (p^2)]'' = -\gamma\ \frac{\delta\Sigma_i\ (p^2)}{p^2 \ln p^2/\Lambda^2} - \frac{3\alpha_i}{4\pi}\ \frac{4\ m_D^3}{(p^2)^2}\,[\ln \frac{p^2}{\Lambda^2}\,]^{\gamma},$$

$$\gamma = \frac{12}{33-2N_F}\ . \tag{5.4}$$

Here the differentiation is with respect to p^2 and we have used relation (3.11b) replacing μ^2 by Λ^2 and $g_s^2(k^2)/4\pi = \pi\gamma\ /\ \ln \frac{k^2}{\Lambda^2}$ has been used. Λ is the usual free mass scale parameter of QCD. Equation (5.4) can be solved analytically. The solution satisfying the appropriate boundary condition

$$\lim_{p^2 \to \infty} \left[p^2 \delta \Sigma_i (p^2) \right]' \to 0$$

is given by

$$\delta \Sigma_i(p^2)_{p^2 \to \infty} = \frac{3\alpha_i}{4\pi} \frac{4 m_D^3}{p^2} \Gamma(1+\gamma)\Gamma(2+\gamma) \sum_{n=0}^{\infty} \left[\frac{\Gamma(n+1-\gamma)}{\Gamma(n+1)\Gamma(n+2)} \right.$$

$$\left. - \frac{(\ln p^2/\Lambda^2)^{\gamma+1}\, \Gamma(n+2)}{\Gamma(\gamma+n+2)\Gamma(\gamma+n+3)} \right] \left[\ln \frac{p^2}{\Lambda^2} \right]^{n+1} \tag{5.5a}$$

$$= \frac{3\alpha_i}{4\pi} \frac{4 m_D^3}{p^2} \Gamma(1+\gamma)\Gamma(2+\gamma) [\Gamma(1-\gamma)\, {}_1F_1(1-\gamma;2;\ln p^2/\Lambda^2$$

$$- \frac{(\ln p^2/\Lambda^2)^{\gamma+1}}{\Gamma(\gamma+2)\Gamma(\gamma+3)}\, {}_2F_2(1,2;\gamma+2,\gamma+3;\ln p^2/\Lambda^2)] \times \ln p^2/\Lambda^2 \,, \tag{5.5b}$$

$$\equiv \frac{3\alpha_i}{4\pi} \frac{4 m_D^3}{p^2} \Gamma(1+\gamma)\Gamma(2+\gamma)\, F(p^2/\Lambda^2) \tag{5.5c}$$

where ${}_mF_n$'s are generalized hypergeometric functions.

We observe that $(\Sigma_u - \Sigma_d)$ will have the correct sign only if $F(p^2/\Lambda^2)$ is negative. Numerical computations show that this does not happen in the range $1 < \ln p^2/\Lambda^2 < 18$ for $N_F = 6$ and in the range $1 < \ln p^2/\Lambda^2 < 16$ for $N_F = 7$ (see Figure 5). If Λ is taken to be 150 MeV, then this corresponds to $(0.4\ \text{GeV})^2 < p^2 < (1410\ \text{GeV})^2$ for $N_F = 6$ and $(0.4\ \text{GeV})^2 < p^2 < (520\ \text{GeV})^2$ for $N_F = 7$.

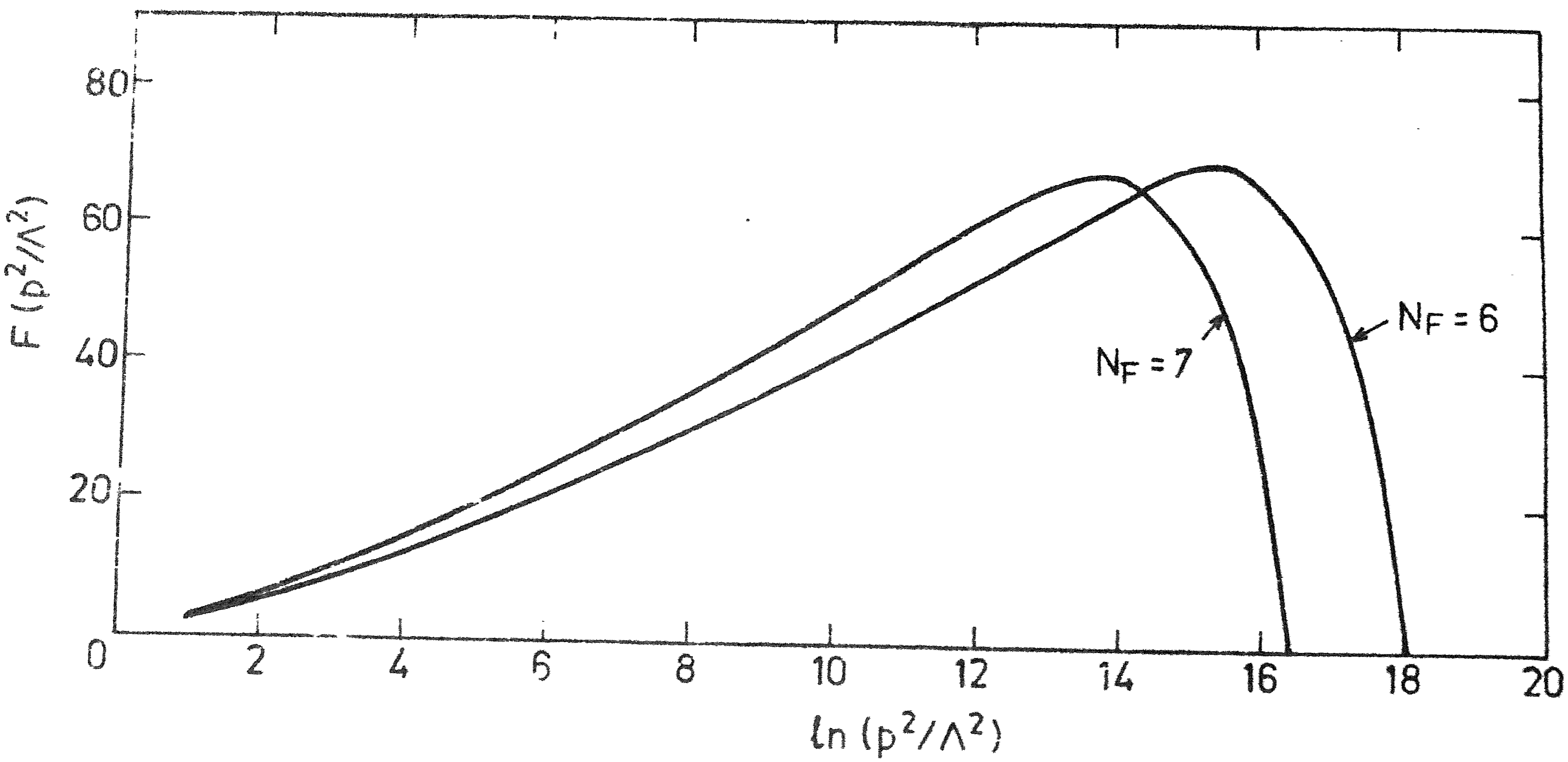

Fig. 5

Thus we see that for reasonable values of momenta we do not get the correct sign of $(\Sigma_u - \Sigma_d)$. However, as in the case of Reference [47], $\delta\Sigma_i(p^2)$ is completely finite and tends to zero as p^2 tends to infinity.

At this stage, one may conclude that the persistence of the wrong sign may be, as enumerated in Reference [47], due to one or more of the following reasons: (i) There is a bare up- and down- quark mass difference. (ii) The result depends on the nonasymptotic part of $\Sigma_o(p^2)$. (iii) Corrections higher than the lowest order (at least in the strong coupling) are important. (iv) Instanton contributions to the kernel are large.

However, recently Chang and Li[62] have also examined this problem and have found the correct sign for $\Sigma_u - \Sigma_d$ within the framework of $SU(3)_c xU(1)_{em}$ with zero current-quark-mass and without invoking instantons. Their work is based on a new approach developed by Chang and Chang[63] to calculate the dynamically generated quark mass in QCD based on the Nambu-Jona-Lasinio[9] mechanism. Chang and Chang have done a renormalization group (RG) analysis of the Nambu-Jona-Lasinio gap equation and found that it is indeed an RG invariant for QCD. With the renormalized two-point Green's function for the quarks given by $(p^2 << m^2)$

$$\Gamma^{(2)}(p) = \tilde{Z}_2^{-1}(\not{p} - m) \tag{5.6}$$

they find that the mass of the quark so generated is, to two loop RG accuracy,

$$m = \Lambda^{(2)} e^{1/6} \tag{5.7}$$

where $\Lambda^{(2)}$ is the free mass scale parameter of QCD evaluated up to two loop. This mass, m, is a RG invariant. Chang and Li have shown the gauge independence of the result (5.7) too. To generate the u-d quark mass difference Chang and Li include the effects of QED as we have done above. To one loop RG accuracy, the degeneracy between u and d quarks is not lifted. To two-loop RG accuracy they find

$$m_u - m_d \simeq -m \frac{e^2}{288\,\pi^2} d_{13} \tag{5.8}$$

where d_{13} represents the genuine two loop constant of the self-energy graphs involving both gluon and photon exchanges and is approximately 64. This gives the right sign and also the right order of magnitude for $m_u - m_d$.

The analysis of Chang and Li has the advantage over ours in the sense that their result is RG and gauge invariant and is meant for the low energy region. Calculation up to two loop RG accuracy gives an additional advantage.

In conclusion, we see that if QCD feedback on electromagnetic self-energy is properly taken into account, it may correctly account for the u-d quark mass difference.

Chapter 6

CONCLUDING REMARKS

In the present thesis, we have studied two types of problems: (i) Dynamical symmetry breaking (DSB) in the Abelian gauge model of Jackiw and Johnson[15] with the help of the gauge technique, and (ii) Applications of dynamical chiral symmetry breaking (XSB) in QCD.

In the first case, the quantity of interest to be calculated was the axial-vector mass to the fermion mass ratio (μ/m). To apply the gauge technique to this problem one needs to know the fermion spectral function (ρ). In view of the complexity of the problem of finding ρ, we chose to work in a sector where μ/m was very small. If we assume that the result for the mass ratio obtained by earlier authors is at least qualitatively true, then this implies working in a sector where the axial-vector to vector coupling constant ratio (g'/g) is very small. In such a case we can hope to expand ρ in terms of these small parameters (unless ρ depends upon these ratios in a nonanalytic way). This seems sensible because, for example, we expect ρ to reduce to the QED ρ smoothly in the limit $g'/g \to 0$. Since the gauge technique is known to work well in the infrared region, precisely where DSB is supposed to occur, we expect it to give reliable results even in the aforementioned sector. To our surprise, we find that this way the result for μ/m

comes out to be extremely small. This does not conform to the results obtained by earlier authors in the above stated sector. Firstly, using the gauge technique, we do not agree with earlier authors that one can use axial-meson propagator with mass $\mu = 0$ to obtain a non-zero fermion mass and then subsequently to obtain $\mu \neq 0$ (see Chapter 2). This seems natural, since unless one takes the perturbation expansion around the new vacuum in which the particles have already become massive, conventional perturbation theory will not reveal the structure of the massive theory, if it is a massive one at all. Secondly, we evaluate μ^2/m^2 with a term in ρ which contains information about μ/m being nonzero, and thus μ/m is determined in a self-consistent way. Our method cannot determine the individual masses (because the mass-scale in the theory is set essentially by the renormalization process, which we are not using explicitly). In particular, we can not say whether the mass m is non-zero.

In the second case, we have applied (dynamical) XSB solutions for the quark propagator to two problems: (a) Anomalous magnetic moments (a.m. m.'s) of light quarks and (b) Up- and down- quark mass difference.

We have seen that in theories in which chiral symmetry breaks dynamically, a.m. m.'s of quarks can be significant, although not appreciably large as one might speculate naively (see Chapter 4). Among the various XSB solutions for the quark propagator that we have examined,

the solution[35] obtained by using the singular form of gluon propagator[49] and the longitudinal part of quark-gluon vertex is the most interesting one. To be consistent, we too have used the singular form of gluon propagator in the calculation. Since the quark propagator which we use is supposed to describe confined quarks (because it has no "mass shell" singularity), we have not used any mass-shell condition which would have otherwise given an additional contribution to a.m.m. term. One interesting aspect of our result for the a.m.m. term is that it has desirable momentum dependence in the infrared as well as in the ultraviolet region, i.e., it vanishes linearly with q (momentum transfer) as $q \to 0$ and inversely with q as $q \to \infty$. This result (for low energy) along with the effective quark mass was used to calculate proton and neutron magnetic moments using regular SU(6) state vectors for baryons. The result thus obtained was in good agreement with experiment. We also calculated spin-dependent potential between a heavy quark and a light antiquark. When this was applied to calculate the spectrum of ground state mesons, we found that the anomalous chromomagnetic moment had no major role. We further found that the contribution of the anomalous chromomagnetic moment in differential cross section for $q\bar{q} \to q\bar{q}$ was insignificant (at high energy) at any angle $\theta \gtrsim 10^{-2}$.

Our result of the calculation of up- and down-quark mass difference comes out with the wrong sign for any reasonable value of momenta, although it is finite and tends to zero as the $(\text{momentum})^2 \to \infty$. (see Chapter 5). We are left with one or more of the following reasons for the persistence of the wrong sign: (i) There is a bare up- and down-quark mass difference. (ii) The result depends on the nonasymptotic part of the quark self-energy function. (iii) Corrections higher than the lowest order (at least in the strong coupling) are important. (iv) Instanton contributions to the kernel are large. Chang and Li[62] have obtained the correct sign (and also the correct order of magnitude) of $m_u - m_d$ from their calculation of masses at low energy up to two-loop renormalization group accuracy. This work narrows down the possible ways to improve our result. It indicates that it may be possible to obtain $m_u - m_d$ correctly by generating the quark masses entirely by color and electromagnetic forces. We feel that a possible way to improve our result may be to analyze the XSB solution of the Schwinger-Dyson equation for the quark propagator in the infrared region using both gluon and photon exchanges, and using longitudinal vertices for the complete ones. The result may be further improved by requiring it to be gauge independent and renormalization group invariant.

We have not shown the gauge independence of any of our results in the thesis. This is a difficult problem and only recently some progress seems to have been made in this direction. Our attempts to use the idea of DSB to extract physically observable results are encouraging to consider further work on it worthwhile.

Appendix A

INTEGRAL EQUATION FOR FERMION SPECTRAL FUNCTION

The more familiar form of spectral function given by

$$S(p) = \int_{m^2}^{\infty} \frac{(\not{p}\rho_1(\mu^2) + m\rho_2(\mu^2))d\mu^2}{p^2 - \mu^2 + i\varepsilon}$$

with $m_o = Z_2 \int m\rho_2(\mu^2)d\mu^2$, $1 = Z_2 \int \rho_1(\mu^2)d\mu^2$ is connected with the form (2.3) by[22]

$$\rho(W) = \varepsilon(W)(W\rho_1(W^2) + m\rho_2(W^2)).$$

Now using Equations (2.4a), (2.4b) and the spectral decomposition for Z_2 in the SD equation for the fermion propagator (Equation (2.12)), one gets

$$\int \rho(W)dW = \int \frac{dW}{\not{p}-W+i\delta}\frac{\rho(W)}{\varepsilon(W)}[\not{p} - \Sigma_A(p,W) - \Sigma_B(p,W)] \qquad \text{(A-1)}$$

where

$$\Sigma_A(p,W) = ig^2 \int (dk)\gamma_\mu \frac{1}{\not{p}-\not{k}-W+i\varepsilon}\gamma_\nu D_A^{\mu\nu}(k) \qquad \text{(A-1a)}$$

$$\Sigma_B(p,W) = ig'^2 \int (dk)\gamma_\mu\gamma_5 \frac{1}{\not{p}-\not{k}-W+i\varepsilon}\gamma_\nu\gamma_5 D_B^{\mu\nu}(k) \qquad \text{(A-1b)}$$

Pole term in axial vertex does not contribute to $\Sigma_B(p,W)$ because gauge boson propagators are in the Landau gauge. Using projection operators $P_\pm = \frac{1}{2} \pm \frac{1}{2}\frac{\not{p}}{\sqrt{p^2}}$, which converts

equation $\not p\, A(p^2) + B(p^2) = 0$ into $\sqrt{p^2}\, A(p^2) + B(p^2) = 0$, Equation (A-1) can be written as

$$\int \frac{dW' \rho(W')}{W-W'+i\delta\varepsilon(W')} \left[W' - \Sigma_A(W,W') - \Sigma_B(W,W')\right] = 0 \,. \qquad \text{(A-2)}$$

On taking imaginary part of Equation (A-2), it becomes

$$-\pi\varepsilon(W)\rho(W)\left[W - \Sigma_A(W,W) - \Sigma_B(W,W)\right] = \int \frac{dW'\rho(W')}{W-W'} \times$$
$$\mathrm{Im}\left[\Sigma_A(W,W') + \Sigma_B(W,W')\right]. \qquad \text{(A-3)}$$

If we define

$$m = \Sigma_A(m,m) + \Sigma_B(m,m) \,, \qquad \text{(A-4)}$$

then we can write

$$\rho(W)\left[W - \Sigma_A(W,W) - \Sigma_B(W,W)\right]$$
$$= \rho(W)\left[W - m + \Sigma_A(m,m) - \Sigma_A(W,W) + \Sigma_B(m,m) - \Sigma_B(W,W)\right].$$

$\rho(W)\left[\Sigma_A(m,m) - \Sigma_A(W,W) + \Sigma_B(m,m) - \Sigma_B(W,W)\right]$ is $O(g^4, g^4, g^2 g^2)$, i.e., of the same order as the transverse corrections which have not been taken into account in any case. Thus, upto these corrections, Equation (A-3) can be written as

$$-\varepsilon(W)\rho(W)\left[W-m\right] = \int \frac{dW'\rho(W')}{\pi(W-W')} \mathrm{Im}\left[\Sigma_A(W,W') + \Sigma_B(W,W')\right]. \qquad \text{(A-5)}$$

Appendix B

IMAGINARY PART OF FERMION-SELF-ENERGY FUNCTION

Fermion-self-energy due to its interaction with B-meson in the Landau gauge, to the lowest order is

$$\Sigma_B(p) = -ig'^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu \gamma_5 \frac{1}{\not p - \not k - m + i\varepsilon} \gamma^\nu \gamma_5 \frac{(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2+i\varepsilon})}{k^2 - \mu^2 + i\varepsilon}$$

$$= +ig'^2 \int \frac{d^4k}{(2\pi)^4} \frac{3m + \not p - 3\not k + \frac{2p.k\not k}{k^2+i\varepsilon}}{[(p-k)^2 - m^2 + i\varepsilon](k^2-\mu^2+i\varepsilon)} \equiv F'(p^2) + \not p\, F(p^2) \tag{B-1}$$

where

$$F' = ig'^2 \int \frac{d^4k}{(2\pi)^4} \frac{3m}{[(p-k)^2 - m^2 + i\varepsilon](k^2-\mu^2+i\varepsilon)} \tag{B-1a}$$

and

$$p^2 F(p^2) = ig'^2 \int \frac{d^4k}{(2\pi)^4} \frac{p^2 - 3p.k + \frac{2(p.k)^2}{k^2+i\varepsilon}}{[(p-k)^2 - m^2 + i\varepsilon](k^2-\mu^2+i\varepsilon)} \tag{B-1b}$$

First consider the integral in (B-1a)

$$F' = ig^2 \int \frac{d^4k}{(2\pi)^4} \frac{3m}{(p_0 - k_0 - \Delta + i\varepsilon)(p_0 - k_0 + \Delta - i\varepsilon)(k_0 - \kappa + i\varepsilon)(k_0 + \kappa - i\varepsilon)},$$

$$\Delta = \{(\vec{p} - \vec{k})^2 + m^2\}^{\frac{1}{2}}; \quad \kappa = (\vec{k}^2 + \mu^2)^{\frac{1}{2}}. \tag{B-2a}$$

We first perform the k_0-integration by closing the contour in

the upper-half-plane:

$$F' = -\frac{g'^2}{(2\pi)^3} \int d^3k \left[\frac{3m}{2\Delta(p_o-\Delta+\kappa)(p_o-\Delta-\kappa+i\varepsilon)} \right.$$

$$\left. + \frac{3m}{(p_o-\Delta+\kappa)(p_o+\Delta+\kappa)(-2\kappa)} \right]. \quad \text{(B-2b)}$$

Equating imaginary parts on both sides we get

$$\mathrm{Im}\, F' = \frac{-g'^2}{4\pi^2} \frac{3m}{4} \int d\Omega \, \vec{k}^2 \, d|\vec{k}| \frac{\delta(p_o - \Delta - \kappa)}{2\,\Delta\kappa} \quad \text{(B-2c)}$$

$$= \frac{-g'^2}{4\pi^2} \frac{3m}{4} \int d\Omega \left. \frac{\vec{k}^2}{2|\kappa(|\vec{k}| - |\vec{p}| \cos\theta) + |\vec{k}|\Delta|} \right|_{|\vec{k}| = \beta},$$

$$\text{(B-2d)}$$

where θ is the angle between $\vec{p}$ and $\vec{k}$, and β is the solution of the equation $p_o - \Delta - \kappa = 0$ in $|\vec{k}|$:

$$\beta = \frac{\lambda^2 |\vec{p}| \cos\theta + (\lambda^4 p_o^2 + 4\mu^2 p_o^2 \vec{p}^2 \cos^2\theta - 4p_o^4\mu^2)^{1/2}}{2(p_o^2 - \vec{p}^2 \cos^2\theta)},$$

$$\lambda^2 = p^2 + \mu^2 - m^2 . \quad \text{(B-2e)}$$

After a straight forward angular integration, Equation (B-2d) becomes

$$\text{Im } F' = \frac{-3g'^2}{16\pi} \; \frac{m}{p^2} \{ p^2-(m+\mu)^2 \}^{1/2} \{ p^2-(m-\mu)^2 \}^{1/2} \theta(p^2-(m+\mu)^2)$$

$$\equiv - \frac{3g'^2}{16\pi} \; \frac{m}{p^2} \phi_\mu (p,m) \; . \tag{B-2f}$$

θ-function on r.h.s. of (B-2f) insured that the expression is real. Integrations in (B-1b) can similarly be done:

$$\text{Im } (p^2 \; F_1) = \text{Im } ig'^2 \int \frac{d^4k}{(2\pi)^4} \; \frac{p^2}{[(p-k)^2-m^2+i\varepsilon](k^2-\mu^2+i\varepsilon)}$$

$$= \frac{-g'^2}{16\pi} \; \phi_\mu (p,m) \; , \tag{B-3}$$

$$\text{Im } (p^2 F_2) = \text{Im } ig'^2 \int \frac{d^4k}{(2\pi)^4} \; \frac{-3 \; p.k}{[(p-k)^2-m^2+i\varepsilon] (k^2-\mu^2+i\varepsilon)}$$

$$= \frac{3g'^2}{32\pi p^2} \; (p^2+\mu^2-m^2) \phi_\mu(p,m), \tag{B-4}$$

$$\text{Im} (p^2 F_3) = \text{Im } ig'^2 \int \frac{d^4k}{(2\pi)^4} \; \frac{2(p.k)^2}{[(p-k)^2-m^2+i\varepsilon](k^2-\mu^2+i\varepsilon)(k^2+i\varepsilon)}$$

$$= \frac{-g'^2}{32\pi p^2 \mu^2} \; [(p^2+\mu^2-m^2)^2 \phi_\mu (p,m) - (p^2-m^2)^2 \phi_o (p,m)], \tag{B-5}$$

where $\phi_o(p,m) = \phi_\mu(p,m)|_{\mu=0}$. Thus,

$$\text{Im } F = \text{Im } (F_1 + F_2 + F_3)$$

$$= \frac{-g'^2}{32\pi p^4} [(p^2+m^2-2\mu^2)\phi_\mu + \mu^{-2}(p^2-m^2)^2 (\phi_\mu - \phi_o)]. \tag{B-6}$$

Substituting Equations (B-2f) and (B-6) in Equation (B-1), we get

$$\text{Im}\ \Sigma_B(\not p, W) = \frac{-g'^2}{32\ \pi p^2} \Big[\Big\{ \frac{\not p}{p^2}\ (p^2+W^2-2\mu^2)+6W \Big\}\ \phi_\mu$$

$$+ \frac{\not p}{p^2}\ \mu^{-2}(p^2-W^2)^2(\phi_\mu - \phi_o) \Big], \qquad \text{(B-7)}$$

where

$$\phi_\mu\ (p, W) = \{(p^2-(W+\mu)^2\}^{1/2}\ \{p^2-(W-\mu)^2\}^{1/2}\ \theta(p^2-(W+\mu)^2),$$

$$\phi_o(p, W) = (p^2-W^2)\ \theta(p^2-W^2)\ . \qquad \text{(B-8)}$$

Taking the limit

$$\lim_{\mu\to 0}\ \mu^{-2}\ (\phi_\mu - \phi_o) = -\{(p^2+W^2)/(p^2-W^2)\}\ \theta(p^2-W^2).$$

Thus,

$$\text{Im}\ \Sigma_B(\not p, W)\Big|_{\mu\to 0} = \frac{-3\ g'^2 W}{16\ \pi p^2}\ (p^2-W^2)\ \theta(p^2-W^2). \qquad \text{(B-9)}$$

Now, from the definition

$$\Sigma_A(p) = -ig^2 \int \frac{d^4k}{(2\pi)^4}\ \gamma^\mu \frac{1}{\not p - \not k - m + i\varepsilon}\ \gamma^\nu\ \frac{\left(g_{\mu\nu} - \dfrac{k_\mu k_\nu}{k^2+i\varepsilon}\right)}{k^2+i\varepsilon}, \qquad \text{(B-10)}$$

it is clear that

$$\text{Im}\ \Sigma_A(\not p, W) = +\frac{3g^2 W}{16\ \pi p^2}\ (p^2-W^2)\ \theta(p^2-W^2). \qquad \text{(B-11)}$$

Appendix C

FERMION SPECTRAL FUNCTION IN QED

Integral equation satisfied by the fermion spectral function in QED is Equation (2.16) with $\delta' = 0$:

$$\omega^2 \varepsilon(\omega)\ (\omega - 1) r^{o'}(\omega) = -\,\delta \int d\omega' r^{o'}(\omega')\, \omega'(\omega + \omega')\ . \qquad \text{(C-1)}$$

Let us substitute

$$\varepsilon(\omega)\, \omega^2\, r^{o'}(\omega) = \omega\, r_1(\omega^2) + r_2(\omega^2)$$

in Equation (C-1). On equating even and odd parts in ω, we get two equations ($\omega^2 \equiv z$)

$$r_2(z) - r_1(z) = -\delta \int_1^z r_2(z')/z'\, dz', \qquad \text{(C-2a)}$$

$$z\, r_1(z) - r_2(z) = -\delta \int_1^z r_1(z')\, dz'\ . \qquad \text{(C-2b)}$$

This pair of equations can be reconverted into hypergeometric equations

$$\left[z(1-z) \frac{d^2}{dz^2} + \{1 - z(3+2\delta)\} \frac{d}{dz} - (1+\delta)^2 \right] r_1(z) = 0, \qquad \text{(C-3a)}$$

$$\left[z(1-z) \frac{d^2}{dz^2} - 2(1+\delta)\, z \frac{d}{dz} - \delta\, (1+\delta) \right] r_2(z) = 0. \qquad \text{(C-3b)}$$

The appropriate solutions, satisfying the boundary conditions embodied in (C-2a) and (C-2b) are

$$r_1(z) = \frac{(z-1)^{-1-2\delta}}{\Gamma(-2\delta)2^{-2\delta}} F(-\delta,-\delta;\ -2\delta;\ 1-z), \tag{C-4a}$$

$$r_2(z) = \frac{z(z-1)^{-1-2\delta}}{\Gamma(-2\delta)2^{-2\delta}} F(-\delta,1-\delta;-2\delta;\ 1-z). \tag{C-4b}$$

Normalization in (C-4a) and (C-4b) is such that

$$r^{o'}(\omega) \to \delta(\omega-1) \text{ as } g \to 0 \ .$$

Thus we get expression (2.17).

Now we shall show that the Goldstone mode can not be realized with a fermion spectral function of the form $r^{o'}(\omega)$. Let us substitute expression (2.21) in Equation (2.11b):

$$\frac{\mu^2}{m^2} = \frac{g'^2 Z_2}{2\pi^2} \iint \frac{\omega^2 r^o(\omega)\ (q^2)^\alpha d\omega dq^2}{(q^2+\omega^2)^2} \tag{C-5}$$

where lim $\alpha \to 1$ will be taken after the integration. We shall use the standard integrals

$$\int_0^\infty F(a,b;c;-x)x^{-s-1}\,dx = \frac{\Gamma(a+s)\Gamma(b+s)\Gamma(c)\Gamma(-s)}{\Gamma(a)\Gamma(b)\Gamma(c+s)}$$

and

$$\int_0^\infty x^{c-1}(x+y)^{-d} F(a,b;c;-x)dx = \frac{\Gamma(a-c+d)\Gamma(b-c+d)\Gamma(c)}{\Gamma(a+b-c+d)\Gamma(d)}$$

$$\times F(a-c+d,b-c+d;a+b-c+d;1-y).$$

This gives $Z_2 = c^{-1}\,[2^{\delta_2}\Gamma(1+\delta_2)]^{-2}$, and when substituted in (C-5), we get

$$\frac{\mu^2}{m^2} = g'^2 \frac{(1+\delta_2)^2}{2\pi^2} \int_0^{\infty} dz\; z^{\alpha} F(2+\delta_2,\; 2+\delta_2;\; 2;\; -z)$$

$$= -g'^2 \frac{1}{2} \left(\frac{\Gamma(1+\delta_2-\alpha)}{\Gamma(1+\delta_2)}\right)^2 \frac{\Gamma(1+\alpha)}{\Gamma(1-\alpha)} \xrightarrow[\alpha\to 1]{} 0\,. \qquad \text{(C-6)}$$

Appendix D

SOLUTION OF THE APPROXIMATE INTEGRAL EQUATION FOR SPECTRAL FUNCTION

(a) Infrared region $((|W|-m) \lesssim \mu)$

To solve Equation (2.16), we write it in the following form:

$$\omega^2(\omega-1)r(\omega) \simeq -\delta\int d\omega' \; r(\omega')\omega'(\omega+\omega')\theta(\omega^2-\omega'^2)$$
$$-\frac{4}{3}\delta' \int_1^\omega d\omega' r(\omega') \frac{(\omega-\omega')^2}{\mu^2/m^2}, \qquad \text{(D-1a)}$$

$$\omega^2(\omega+1)r(-\omega) \simeq \delta\int d\omega' \; r(-\omega')\omega'(\omega+\omega')\theta(\omega^2-\omega'^2)$$
$$+\frac{4}{3}\delta' \int_1^\omega d\omega' \; r(-\omega') \frac{(\omega-\omega')^2}{\mu^2/m^2}, \qquad \text{(D-1b)}$$

where $\omega > 0$ and in the second term on r.h.s. we have put $\omega+\omega' \simeq 2$, etc. Equations (D-1a) and (D-1b) can be combined to write

$$\omega^2(\omega r_+(\omega) - r_-(\omega)) \simeq -\delta\int d\omega' \; r_-(\omega')\omega'(\omega+\omega') \; \theta(\omega^2-\omega'^2)$$
$$-\frac{4}{3}\delta' \int_1^\omega d\omega' \; r_-(\omega') \frac{(\omega-\omega')^2}{\mu^2/m^2}, \qquad \text{(D-2a)}$$

$$\omega^2(\omega r_-(\omega) - r_+(\omega)) \simeq -\delta\int d\omega' \; r_+(\omega')\omega'(\omega+\omega') \; \theta(\omega^2-\omega^2)$$
$$-\frac{4}{3}\delta' \int_1^\omega d\omega' r_+(\omega') \frac{(\omega-\omega')^2}{\mu^2/m^2}. \qquad \text{(D-2b)}$$

Let us substitute Equation (2.18) in Equations (D-2a) and (D-2b):

$$\omega \Delta r_+'(\omega) - \Delta r_-'(\omega) + 2\delta \int_1^{\omega} d\omega' \, \Delta r_-'(\omega')$$

$$\simeq -\frac{4}{3}\delta' \frac{m^2}{\mu^2} \int_1^{\omega} d\omega' \, r_-^{o'}(\omega')(\omega-\omega')^2 , \tag{D-3a}$$

$$\omega \Delta r_-'(\omega) - \Delta r_+'(\omega) + 2\delta \int_1^{\omega} d\omega' \, \Delta r_+'(\omega')$$

$$\simeq -\frac{4}{3}\delta' \frac{m^2}{\mu^2} \int_1^{\omega} d\omega' \, r_+^{o'}(\omega')\,(\omega-\omega')^2 . \tag{D-3b}$$

Next we substitute

$$\Delta r_+'(\omega) = u_1(\omega^2)\, r_+^{o'}(\omega) \qquad \Delta r_-'(\omega) = v_1(\omega^2)\, r_-^{o'}(\omega)$$

in Equations (D-3a) and (D-3b), differentiate twice and use the integral equations satisfied by $r_\pm^{o'}$; then one can see that a possible solution is

$$u_1(\omega^2) = v_1(\omega^2) = -\frac{\delta'}{3}\, \frac{m^2}{\mu^2}\, \frac{\Gamma(-2\delta)}{\Gamma(2-2\delta)}\, (\omega^2-1)^2 + a \tag{D-4}$$

up to leading order in (ω^2-1). When substituted back in Equations (D-3a) or (D-3b), the constant of integration 'a' turns out to be zero.

(b) Asymptotic region

Let us perform the following break:

$$r(\omega) = \varepsilon(\omega)\,(\omega r_e(\omega^2) + r_o(\omega^2)). \qquad \text{(D-5)}$$

Substituting Equation (D-5) in Equation (2.20) we have

$$\omega^2[\omega^2\, r_e(\omega^2) - r_o(\omega^2) + \omega\,(r_o(\omega^2) - r_e(\omega^2))]$$

$$= -\delta_2 \left(\int_{-\omega}^{-1} + \int_{1}^{\omega} \right) d\omega'\omega'\; \varepsilon(\omega')\; (\omega r_o(\omega'^2) + \omega'^2\, r_e(\omega'^2))$$

$$+ \delta' \left(\int_{-(\omega+\mu/m)}^{-\omega} - \int_{\omega-\mu/m}^{\omega} \right) d\omega'\; r(\omega')(\omega'(\omega+\omega') + \frac{(\omega+\omega')(\omega^2-\omega'^2)^2}{6\,\omega\,\mu^2/m^2}$$

$$\simeq -\delta_2 \int_{1}^{\omega^2} d\omega'^2\; (\omega r_o(\omega'^2) + \omega'^2\, r_e(\omega'^2))$$

$$-\delta'\, \frac{\mu}{m}\, \frac{22}{9}\, \omega^2\, (\omega r_e(\omega^2) + r_o(\omega^2)). \qquad \text{(D-6)}$$

On equating odd and even parts in ω, Equation (D-6) is equivalent to two equations. Again we break $r(\omega)$ as $r^o(\omega) + \Delta r(\omega)$, where $r^o(\omega)$ is the solution of Equation (D-6) with $\mu/m = 0$ and is given by Equation (2.21). Writing

$$\Delta\, r_e(\omega) = u_2(\omega^2)\; r_e^o(\omega^2), \quad \Delta\, r_o(\omega) = v_2(\omega^2) r_o^o(\omega^2)$$

and expanding r_e^o and r_o^o asymptotically on both sides, we find that

$$u_2(\omega^2) \simeq -\frac{44}{9}\,\delta'(\mu/m)\,\omega^{-2} + b_1 \qquad \text{(D-7a)}$$

$$v_2(\omega^2) \simeq \frac{44}{9}\,\delta'(\mu/m)\,\delta_2\,\frac{\ln\omega^2+1}{\omega^2} + b_2. \qquad \text{(D-7b)}$$

Substituting back in the integral equations, we find $b_1=b_2=0$.

To see how far these asymptotic behaviors are applicable in the low-energy region, we proceed as follows. Consider a region $|\omega|\simeq\lambda$ such that $(\lambda-1)\ll 1$ and $\mu/m \ll (\lambda-1)$ (one can think of $\mu/m \sim O[(\lambda-1)^2]$). In this region we have, from Equation (2.20),

$$\omega^2(\omega-1)\,r(\omega) + \delta_2\int d\omega'\,r(\omega')\omega'(\omega+\omega')\,\theta(\omega^2-\omega'^2)$$

$$\simeq -2\delta'\left(\int_1^{\omega} - \int_1^{\omega-\mu/m}\right) d\omega'\left(1+\frac{2}{3}\frac{m^2}{\mu^2}(\omega-\omega')^2\right)r(\omega') \qquad \text{(D-8a)}$$

and

$$\omega^2(\omega+1)\,r(-\omega) - \delta_2\int d\omega' r(-\omega')\omega'(\omega+\omega')\theta(\omega^2-\omega'^2)$$

$$\simeq -2\delta'\left(\int_1^{\omega+\mu/m} - \int_1^{\omega}\right)d\omega'\,r(-\omega')\left(1+\frac{2}{3}\frac{m^2}{\mu^2}(\omega-\omega')^2\right). \qquad \text{(D-8b)}$$

It is clear from Equations (D-8a) and (D-8b) that the contribution to $r(\pm\omega)$ from the integral on r.h.s., is down by a factor of $[(\mu/m)/(\omega-1)]\,\delta'/\delta_2$ compared to the contribution from the l.h.s. The solution of Equations (D-8a) and (D-8b) with r.h.s. equal to zero is given by expression (2.21). Proceeding by the now familiar way, we write $r(\omega) = r^0(\omega) + \Delta r(\omega)$ in Equation (D-8a). Then in terms of even and odd functions

defined as in Equation (2.15'), (D-8a) reduces to

$$\omega\,\Delta r_+(\omega) - \Delta r_-(\omega) + 2\delta_2 \int_1^\omega d\omega'\,\Delta r_-(\omega')$$

$$\simeq -2\delta'\left(\int_1^\omega - \int_1^{\omega-\mu/m}\right)d\omega'\left(1+\frac{2}{3}\frac{m^2}{\mu^2}(\omega-\omega')^2\right)r^o(\omega')$$

$$-2\delta'\left(\int_1^{\omega+\mu/m} - \int_1^{\omega}\right)d\omega'\left(1+\frac{2}{3}\frac{m^2}{\mu^2}(\omega-\omega')^2\right)r^o(-\omega')$$

$$\simeq -c2\delta' \int_{\omega-\mu/m}^{\omega} d\omega'\left(1+\frac{2}{3}\frac{m^2}{\mu^2}(\omega-\omega')^2\right)\frac{\left(\frac{\omega'-1}{2}\right)^{-1-2\delta_2}}{\Gamma(-2\delta_2)}$$

$$\simeq -c\,\frac{22}{9}\,\frac{\delta'}{\Gamma(-2\delta_2)}\,\frac{\mu}{m}\left(\frac{\omega-1}{2}\right)^{-1-2\delta_2}. \qquad \text{(D-9a)}$$

Similarly, from Equation (D-8b), we get

$$\omega\Delta r_-(\omega) - \Delta r_+(\omega) + 2\delta_2\int_1^\omega d\omega'\,\Delta r_+(\omega') \simeq -c\,\frac{22}{9}\,\frac{\delta'}{\Gamma(-2\delta_2)} \times \frac{\mu}{m}\left(\frac{\omega-1}{2}\right)^{-1-2\delta_2} \qquad \text{(D-9b)}$$

With the substitutions

$$\Delta r_+(\omega) = u_2'(\omega^2)\,r_+^o(\omega) \qquad \Delta r_-(\omega) = v_2'(\omega^2)\,r_-^o(\omega)$$

in Equations (D-9a) and (D-9b), one finds that a possible solution is

$$u_2'(\omega^2) = v_2'(\omega^2) \simeq -\frac{44}{9}\,\delta'(\mu/m)\,(\omega^2-1)^{-1}. \qquad \text{(D-10)}$$

Comparing Equations (D-7a) and (D-7b) with Equation (D-10), one observes that one can identify $u_2'(\omega^2)$ with $u_2(\omega^2)$ in the corresponding limit, but the same is not true for $v_2'(\omega^2)$ and $v_2(\omega^2)$. Thus it seems reasonable to write

$$\Delta r_+(\omega) \simeq u_2'(\omega^2)\ r_+^o(\omega) \qquad \lambda \lesssim |\omega| \lesssim \infty . \qquad (2.23a)$$

On the other hand, $v_2'(\omega^2)$ and $v_2(\omega^2)$ seem to be the limiting cases of some transcendental function. We shall write an expression for $\Delta r_-(\omega)$ which behaves as $v_2'(\omega^2) r_-^o(\omega)$ and $v_2(\omega^2)\ r_-^o(\omega)$ in the respective limits:

$$\Delta r_-(\omega) \simeq -\frac{44}{9}\delta'\ (\mu/m)\ (\omega^2-1)^{-1}\omega\ r_+^o(\omega) \qquad \lambda \lesssim |\omega| \lesssim \infty . \qquad (2.23b)$$

Appendix E

APPROXIMATE INTEGRAL EQUATION FOR SPECTRAL FUNCTION WHEN $\mu/(W-m) \ll 1$

We shall simplify expression for $\text{Im}\,\Sigma_B$ [Equation (2.14b)] in the above region neglecting terms of order $\frac{\mu^2}{m(W-m)}$ and μ^2/m^2. We recall that $|W|, |W'| \geq m$. We also note that in the full integral Equation (2.14), $\text{Im}\,\Sigma_B(W,W')$ appears along with the spectral function $\rho(W')$ which multiplies it. $\rho(W')$ is concentrated mainly at $W'=m$. Thus terms like $\frac{\mu^2}{W^2-W'^2}$, $\frac{\mu^4 W'^2}{(W^2-W'^2)^3}$, etc. appearing in the expansion of $\text{Im}\,\Sigma_B$ can be approximated by $\frac{\mu^2}{W^2-m^2}$, $\frac{\mu^4 m^2}{(W^2-m^2)^3}$, etc. With these approximations, simple algebra yields

$$\text{Im}\,\Sigma_B(W,W') \simeq -\frac{g'^2}{32\pi W^3}\Big[6\,WW'\,(W^2-W'^2)\,\theta(W^2-(W'+\mu)^2)$$

$$+\frac{(W^2-W'^2)^3}{\mu^2}\{\theta(W^2-(W'+\mu)^2)-\theta(W^2-W'^2)\}\Big] \qquad \text{(E-1)}$$

When Equation (E-1) (along with Equation (2.14a)) is substituted in Equation (2.14), in terms of dimensionless quantities, Equation (2.20) is obtained.

Appendix F

SOLUTION OF THE GAP EQUATION USING ASYMPTOTIC FREEDOM

Following Lane's work[10], we shall solve the gap equation (3.10) using the fact that the QCD is asymptotically free.

We define a nonzero mass, m_R, in terms of which a XSB solution for the quark propagator may be written

$$S^{-1}(p,\mu,m_R,g_R) = \not{p}\ F(p,\mu,m_R,g_R) - m_R\ G(p,\mu,m_R,g_R). \tag{F-1}$$

In terms of (F-1), the gap equation (3.10) reads

$$2m_R\ G(p,\mu,m_R,g_R)(\gamma_5)_{ab} = \int (dk)\{\gamma_5, S(k,\mu,m_R,g_R)\}_{cd}$$
$$K_{ac,db}(p,k,0;\mu,m_R,g_R)\ . \tag{F-2}$$

With κ as a dimensionless parameter used to scale the momentum p, the solutions to the renormalization-group (RG) equations for F and G turn out to be

$$F(\kappa p,\mu,m_R,g_R) = \exp\left(-\int_{g_R}^{g(\kappa)} \frac{dx\,\gamma(x)}{\beta(x)}\right) F(p,\mu,m(\kappa),\ g(\kappa)), \tag{F-3a}$$

and

$$G(\kappa p,\mu,m_R,g_R) = \exp\left(-\int_{g_R}^{g(\kappa)} \frac{dx}{\beta(x)}\left[\gamma(x)+\gamma_m(x)\right]\right) G(p,\mu,m(\kappa),g(\kappa)), \tag{F-3b}$$

whereas for the kernel the solution is

$$K(\kappa p,\kappa k,0;\mu,m_R,g_R) = \kappa^{-2} \exp\left(-2 \int_{g_R}^{g(\kappa)} \frac{dx\,\gamma(x)}{\beta(x)}\right) K(p,k,0;\mu,m(\kappa),g(\kappa)), \tag{F-4}$$

where

$$g^2(\kappa) \underset{\kappa\to\infty}{\sim} (b \ln\kappa)^{-1} + O\left(\frac{\ln\ln\kappa}{(\ln\kappa)^2}\right) \tag{F-5}$$

and

$$m(\kappa) = \frac{m_R}{\kappa} \exp\left(-\int_{g_R}^{g(\kappa)} \frac{dx\,\gamma_m(x)}{\beta(x)}\right)$$

$$\underset{\kappa\to\infty}{\sim} \frac{m_R}{\kappa} (\ln\kappa)^{-c/b}. \tag{F-6}$$

For QCD with N_F quarks, functions β, γ_m, and γ in the Landau gauge (for small g_R) are given by

$$\beta(g_R) \cong -\frac{b}{2} g_R^3 , \quad b = \frac{33-2\,N_F}{24\pi^2} \tag{F-7a}$$

$$\gamma_m(g_R) \cong c\, g_R^2 , \quad c = \frac{1}{2\pi^2} \tag{F-7b}$$

$$\gamma(g_R) \cong f\, g_R^4 . \tag{F-7c}$$

Equation (F-2) can be rewritten in the form

$$G(\kappa p,\mu, m_R, g_R)$$

$$= \kappa^4 \int (dk) \frac{G(\kappa k,\ldots,g_R)\, K(\kappa p,\kappa k,\ldots g_R)}{\kappa^2 k^2 F^2(\kappa k,\ldots,g_R) - m_R^2 G^2(\kappa k,\ldots g_R)} , \tag{F-8}$$

where

$$K(p,k,q;\mu,m_R,g_R) = \frac{1}{4}\,(\gamma_5)_{cd}\,K_{ac,db}(p,k,\ldots)(\gamma_5)_{ba}. \quad \text{(F-9)}$$

From Equations (F-3)-(F-9), it follows that $G(p,\mu,m(\kappa),g(\kappa))$ satisfies the same integral equation as $G(p,\mu,m_R,g_R)$:

$$G(p,\mu,m(\kappa),g(\kappa)) = \int (dk)\ \frac{G(k,\mu,m(\kappa),g(\kappa))\ K(p,k,0;\mu,m(\kappa),g(\kappa))}{k^2F^2(k,\ldots,g(\kappa))-m^2(\kappa)\ G^2(k,\ldots,g(\kappa))}. \quad \text{(F-10)}$$

We expect the m-even functions F and G to behave in the limit $m(\kappa) \to 0$ as

$$F(p,\mu,m(\kappa),g(\kappa)) = F(p,\mu,0,g(\kappa)) + \frac{1}{2}\,m^2(\kappa)\ \left(\frac{\partial^2 F}{\partial m^2}\right)_{m=0} + O(m^4(\kappa)\ \ln m^2(\kappa)) \quad \text{(F-11a)}$$

$$G(p,\mu,m(\kappa),\ g(\kappa)) = G(p,\mu,0,g(\kappa)) + O(m^2(\kappa)\ \ln m^2(\kappa)). \quad \text{(F-11b)}$$

A similar statement is true for the kernel as well

$$K(p,k,0;\mu,m(\kappa),g(\kappa)) = K(p,k,0;\mu,0,g(\kappa)) + O(m^2(\kappa)). \quad \text{(F-12)}$$

Thus for $\kappa \to \infty$, Equation (F-10) becomes

$$G(p,\mu,0,g(\kappa)) = \int (dk)\ \frac{G(k,\mu,0,\ g(\kappa))}{k^2F^2(k,\mu,0,g(\kappa))}\ K(p,k,0;\mu,0,g(\kappa)). \quad \text{(F-13)}$$

Taking advantage of asymptotic freedom, we approximate the kernel (in the Landau gauge) by

$$K_{ac,db}(p,k,0;\mu,0,g(\kappa)) = i^3 g^2(\kappa)\ C_2(3)(\gamma^\lambda)_{ac}$$

$$\left[\frac{(k-p)_\lambda (k-p)_\nu/(k-p)^2 - g_{\lambda\nu}}{(k-p)^2 + i\varepsilon}\right](\gamma^\nu)_{db}$$

$$+\ O(g^4(\kappa)), \tag{F-14a}$$

or, from Equation (F-9),

$$K(p,k,0;\mu,0,g(\kappa)) = 3\ i^3 g^2(\kappa)\ C_2(3)\ \frac{1}{(k-p)^2} + O(g^4(\kappa)). \tag{F-14b}$$

Our final approximation will be

$$F^2(k,\mu,0,g(\kappa)) = 1 + O(g^2(\kappa)). \tag{F-15}$$

The integral equation (F-13) ultimately becomes (letting $p/\mu \to p$ and $k/\mu \to k$)

$$G(p,g(\kappa)) = -3ig^2(\kappa)\ C_2(3) \int (dk)\ \frac{G(k,g(\kappa))}{k^2(k-p)^2}\ . \tag{F-16}$$

To solve Equation (F-16), we first rewrite it in euclidean form via a Wick rotation and then operate on both sides by $\Box$. Using the identities $\Box \dfrac{1}{(p-k)^2} = -4\pi^2\delta^4(p-k)$ and $\Box G = 4(p^2 G)''$, where the derivative is with respect to p^2, Equation (F-16) can be written in the form

$$[\, p^2 G(p,g(\kappa))]'' = -\frac{3g^2(\kappa)\ C_2(3)}{16\pi^2}\, G(p,g(\kappa))/p^2 . \tag{F-17}$$

The solutions of this equation are subject to the boundary conditions

$$\lim_{p^2 \to \infty} \left(p^2 \frac{d}{dp^2} + 1\right) G = 0, \tag{F-18a}$$

$$\lim_{p^2 \to 0} p^4 \frac{d}{dp^2} G = 0. \tag{F-18b}$$

We define

$$\nu = +[1 - 3g^2(\kappa)\ C_2(3)/4\pi^2\ ]^{1/2}$$

$$\cong 1 - c/(b \ln \kappa), \tag{F-19}$$

where, from Equations (F-7), the ratio c/b is

$$\frac{c}{b} = \frac{12}{33 - 2\ N_F} . \tag{F-20}$$

The solutions to Equation (F-17), then, are

$$G_+(p,g(\kappa)) = a_+\, p^{-1+\nu} \underset{\kappa \to \infty}{\cong} a_+ \left(\frac{\ln \kappa}{\ln \kappa p}\right)^{c/b}, \tag{F-21a}$$

and

$$G_-(p,g(\kappa)) = a_- p^{-1-\nu} \underset{\kappa \to \infty}{\cong} \frac{a_-}{p^2} \left(\frac{\ln \kappa p}{\ln \kappa}\right)^{c/b} . \tag{F-21b}$$

Also $G(\kappa p)$ is given, asymptotically, by Equation (F-3b) as

$$G(\kappa p, \mu, m_R, g_R) \cong [g^2(\kappa)]^{c/b} [G(p, \mu\, 0, g(\kappa)) + O(m^2(\kappa) \ln m^2(\kappa))]. \tag{F-22}$$

Combining Equations (F-21a) and (F-22), we get

$$G_+(\kappa p, \mu, m_R, g_R) \cong a_+ (\ln \kappa p)^{-c/b} + O(\kappa^{-2} (\ln \kappa)^B).$$

Here, B is an undetermined constant. Similarly, combining Equations (F-21b) and (F-22), we get

$$G_-(\kappa p, \mu, m_R, g_R) \cong \frac{a_-}{p^2} \left[\frac{\ln \kappa p}{(\ln \kappa)^2}\right]^{c/b} + O(\kappa^{-2} (\ln \kappa)^B). \tag{F-23}$$

Since this expression can be a function of κ and p only through their product, a_- must have the asymptotic form (up to a constant)

$$a_- \sim (\ln \kappa)^{2c/b} \kappa^{-2} \tag{F-24a}$$

$$\sim (g^2(\kappa))^{-2c/b} \exp[-2/b\ g^2(\kappa)]. \tag{F-24b}$$

Accordingly, G_- will be written as

$$G_-(p, \mu, m_R, g_R) \underset{\kappa = p/m_R \to \infty}{\sim} \kappa^{-2} (\ln \kappa)^{c/b} + O(\kappa^{-2} (\ln \kappa)^B). \tag{F-25}$$

So far as we can tell from the BS equation alone, the error in G_- is of the same order as the solution itself.

Firstly, we note from Equation (F-22) that the asymptotic behavior $G_+(p) \sim (\ln p)^{-c/b}$ exactly corresponds

to what we learn from a straight forward RG analysis, and does not differ from the case in which quarks have nonzero bare mass and G(p) does not have purely dynamical origin. Thus we expect G_-, which has a softer asymptotic behavior, to be the correct solution. This can happen if it corresponds to G arising solely from dynamics and is proportional, through the WT identity, to the bound-state vertex function $\rho(p,p+q)$ (at $q = 0$). To show this, we write

$$iS(p)\rho(p,p+q)\,\gamma_5\, iS(p+q)\,\delta^{ab}$$

$$= \frac{Z_A}{Z_2} \int d^4x\, e^{ip\cdot x} \langle \pi^a(q)| T[\Psi(x)\,\bar{\Psi}(0)\;\lambda^b]|0\rangle, \qquad \text{(F-26)}$$

where trace over color space is understood. We may learn the asymptotic behavior of ρ, and hence of G, by taking the limit $|p^2| \to \infty$, $(p+q)^2/p^2 \to 1$. Using operator-product expansion, one readily deduces

$$iS(p)\rho\,(p,p+q)\,\gamma_5\, iS(p+q)\;\delta^{ab}$$

$$\sim U(p,\mu,m_R,g_R)\langle \pi^a(q)|\,\bar{\Psi}(0)\gamma_5\,\tfrac{1}{2}\,\lambda^b\Psi(0)|\,0\rangle. \qquad \text{(F-27)}$$

Nonleading terms in (F-27) correspond to operators with canonical dimension four or more, and are expected to be down by approximately a factor of p relative to the exhibited leading term.

Standard RG analysis of the Wilson coefficient function U gives

$$U(\kappa p, \mu, m_R, g_R) = \kappa^{-4} \exp\{ \int_{g_R}^{g(\kappa)} \frac{dx}{\beta(x)} [\gamma_{\bar{\psi}\gamma_5\lambda^b\psi}(x) + \gamma(x)]\}$$

$$\times\ U(p, \mu, m(\kappa), g(\kappa)), \qquad \text{(F-28a)}$$

while from Equations (F-1), (F-3) and (F-6), we know that

$$S^{-1}(\kappa p, \mu, m_R, g_R) = \kappa \exp[- \int_{g_R}^{g(\kappa)} \frac{dx\,\gamma(x)}{\beta(x)}]$$

$$\times\ S^{-1}(p, \mu, m(\kappa), g(\kappa)). \qquad \text{(F-28b)}$$

Finally from the WT identity of the renormalized axial vertex (in the cutoff theory)

$$q^{\lambda} \Gamma^{a}_{5\lambda} = -2m_o(\Lambda)\, Z_A Z_D^{-1} \Gamma^{a}_{5} - Z_A Z_2^{-1}(\gamma_5 \frac{\lambda_a}{2} S^{-1} + S^{-1} \gamma_5 \lambda_{a/2}),$$

where Z_D is the renormalization constant appropriate to Γ^{a}_{5} and $m_o \rightarrow 0$ as $\Lambda \rightarrow \infty$, we see that $m_o(\Lambda)\, Z_A/Z_D = m_R(Z_m Z_A/Z_D)$ and Z_A/Z_2 are cutoff-independent. This implies that

$$\gamma_{\bar{\Psi}\gamma_5\lambda^b\Psi} \equiv \mu \frac{\partial}{\partial\mu} \ln Z_D = \gamma + \gamma_m . \qquad \text{(F-29)}$$

It follows that

$$G(p, \mu, m_R, g_R) \propto \lim_{q \rightarrow 0} \rho\,(p, p+q)$$

$$\widetilde{\kappa = p/m_R \rightarrow \infty}\ \kappa^{-2} \exp[\int_{g_R}^{g(\kappa)} \frac{dx\gamma_m(x)}{\beta(x)}]$$

$$\approx\ \kappa^{-2}(\ln\kappa)^{c/b}, \qquad \text{(F-30)}$$

which is precisely the G_ solution. Moreover, we now know that the error in this solution is $O(\kappa^{-3}(\ln\kappa)^B)$.

Appendix G

CHIRAL-SYMMETRY BREAKING SOLUTION FOR THE QUARK PROPAGATOR USING INFRARED PROPERTIES OF QCD

Here we shall derive results (3.16) following Reference [35]. We start with the SD equation (3.13) satisfied by the quark propagator (3.14). The unrenormalized axial-gauge gluon propagator $D_{\mu\nu}(k)$ will be written as a sum of an infrared singular part given by (3.12) and an infrared finite part. For $S(p-k)\Gamma_\nu(p-k,p)\,S(p)$, longitudinal part given by Equation (3.15) will be used. With these substitutions, Equation (3.13) takes the form

$$1 = \not{p}(\not{p}\,F+G) - g_o^2\, C_2 \int (dk)\gamma_\mu D_{\mu\nu}(k)\left\{ \frac{F+F'}{2}\gamma_\nu + \left(\frac{F-F'}{2}\right)\frac{2\not{p}'\gamma_\nu\not{p}+(p'^2+p^2)\gamma_\nu}{p^2-p'^2} + (G-G')\,\frac{\gamma_\nu\not{p}+\not{p}'\gamma_\nu}{p^2-p'^2}\right\}$$

$$+ X_F + \not{p}\,X_G + \not{n}\text{ terms.} \qquad \text{(G-1)}$$

In Equation (G-1) X_F and X_G are contributions of $\Gamma^{(T)}$. From Equation (G-1) we can write equations for F and G separately which contain I and H respectively with (p.n) as coefficients. We shall specialize to the gauge p.n=0 wherein H and I can be dropped and equations for F and G take the simple forms

$$1 = p^2F - g_o^2C_2 \int (dk)\{ \frac{F+F'}{2}\delta_{\mu\nu} + \frac{F - F'}{p^2-p'^2}[\frac{k^2}{2}\delta_{\mu\nu}+ p'_\mu\ p_\nu+p'_\nu\ p_\mu]\}$$

$$x\ D_{\mu\nu}(k) + X_F \qquad \text{(G-2a)}$$

$$= p^2F+g_o^2C_2Z(M)A\ M^2\int(dk)\ \frac{1}{k^4}\{(F+F')\ (1+ \frac{k^2n^2}{2(k.n)^2})$$

$$+ (F-F')[\frac{k^2+2p^2}{2p.k-k^2} + \frac{n^2(2p.k-k^2)}{2(k.n)^2}]\} + Y_F, \qquad \text{(G-2b)}$$

$$0 = p^2G-g_o^2C_2\int(dk)\ \frac{G - G'}{p^2-p'^2}[\delta_{\mu\nu}(p^2-p.p')+ p_\mu p'_\nu + p_\nu p'_\mu]D_{\mu\nu}(k)+p^2X_G$$

$$\text{(G-3a)}$$

$$= p^2G+g_o^2C_2Z(M)A\ M^2\int\ (dk)\ \frac{1}{k^4}\ (G-G')[1 + \frac{k^2n^2}{2(k.n)^2} + \frac{k^2+2p^2}{2p.k-k^2}$$

$$+ \frac{n^2(2p.k-k^2)}{2(k.n)^2}] + Y_G. \qquad \text{(G-3b)}$$

The functions Y_F and Y_G are X_F and X_G plus the contributions due to nonsingular part of $D_{\mu\nu}$.

First consider the following integral

$$I_1 \equiv \int \frac{d^4k}{k^4} F\ ((p-k)^2)[\ 1\ + \frac{1}{2}\ \ \frac{k^2n^2}{(k.n)^2}]. \qquad \text{(G-4)}$$

We shall assume that F is a well-defined function so that I_1 is finite. To deal with the singularity arising from k^{-4} we shall introduce a regulator prescription defined by the replacement[36]

$$1/k^4 \rightarrow \lim_{\varepsilon\rightarrow 0}\ (1/k^2)\ (1/(k^2+\varepsilon^2)). \qquad \text{(G-5)}$$

Singularity arising due to the use of axial gauge will be dealt with, by using the principal value prescription[64]

$$\frac{1}{(k.n)^{\beta}} \rightarrow \lim_{\varepsilon' \rightarrow 0} \frac{1}{2} \left[\frac{1}{(k.n+i\,\varepsilon')^{\beta}} + \frac{(-1)^{\beta}}{(-k.n+i\,\varepsilon')^{\beta}}\right]. \qquad \text{(G-6)}$$

This prescription, in the present case, amounts to the replacement $\int_0^{\pi} \sin\theta \, d\theta/\cos^2\theta \rightarrow -2$. With these prescriptions, and after shifting the variable and angular integration, integral (G-4) becomes

$$I_1 \rightarrow \pi^2 \int_0^{\infty} dk^2 F(k^2) \left[\frac{((k^2+p^2+\varepsilon^2)^2-4k^2p^2)^{1/2}-|k^2-p^2|}{2p^2\varepsilon^2} - \frac{1}{2p^2} - \frac{1}{((k^2+p^2+\varepsilon^2)^2-4k^2p^2)^{1/2}}\right] \qquad \text{(G-7)}$$

Here it can be observed that the order of taking the limit $\varepsilon \rightarrow 0$ and performing the integration is important. Thus if the limit is taken after the integration (for F=1 in (G-4), ignoring ultraviolet behavior for the time being)

$$I_2 \equiv \pi^2 \int_0^{\infty} dk^2 \left[\frac{\{(k^2+p^2+\varepsilon^2)^2-4k^2p^2\}^{1/2}-|k^2-p^2|}{2p^2\varepsilon^2} - \frac{1}{2p^2} - \frac{1}{\{(k^2+p^2+\varepsilon^2)^2-4k^2p^2\}^{1/2}}\right] = 0$$

whereas if the limit is taken before the integration

$$I_2 \rightarrow -\frac{\pi^2}{p^2} \int_0^{p^2} dk^2 = -\pi^2.$$

This last result would have been obtained if the variable in (G-4) were shifted without introducing ε-regulator prescription and the integration performed. We conclude that this difference is due to the presence of a δ-function in the original integral (G-7) if the limit $\varepsilon \to 0$ is taken before the integration, i.e.,

$$I_1 = \lim_{\kappa \to 0} \Big[-\frac{\pi^2}{p^2} \int_0^{p^2-\kappa^2} dk^2 \; F(k^2) + \pi^2 \int_{p^2-\kappa^2}^{p^2+\kappa^2} dk^2 \delta(k^2-p^2) \times F(k^2) \Big] . \qquad \text{(G-8)}$$

That the r.h.s. of Equations (G-4) and (G-8) are same can be checked by considering some simple algebraic functions for F. Thus if, for example, one chooses $F((p-k)^2)=[(p-k)^2-p^2]/[(p-k)^4 + p^4]$ so that the integral (G-4) is well-defined in the infrared region of integration (and prescription (G-5) is not needed), then the δ-function part of (G-8) does not contribute indeed as it should.

However, relation (G-8) can not be applied when F = constant. The reason is that in this case the original integral (G-4) is logarithmically divergent in the radial variable while the angular integration of the bracketed term is zero (Equation (G-8) takes only the latter into account). To do this kind of integral we shall use dimensional regularization in addition to the two prescriptions mentioned earlier:

$$I_1' \equiv \int \frac{d^4k}{k^4} \left[1 + \frac{1}{2} \quad \frac{k^2n^2}{(k.n)^2}\right]$$

$$\to \lim_{\varepsilon \to 0} \int \frac{d^{2\omega} k}{k^2(k^2+\varepsilon^2)} \left[1 + \frac{1}{2} \quad \frac{k^2n^2}{(k.n)^2}\right], 2\omega < 4. \qquad \text{(G-9)}$$

Second term can be written as[64]

$$\frac{n^2}{2} \; PV \int \frac{d^{2\omega} k}{(k^2+\varepsilon^2)(k.n)^2} = - \frac{2\omega-2}{2} \int \frac{d^{2\omega}k}{k^2(k^2+\varepsilon^2)} \, . \qquad \text{(G-10)}$$

From Equations (G-9) and (G-10), we get

$$I_1' = \frac{4-2\omega}{2} \int \frac{d^{2\omega}k}{k^2(k^2+\varepsilon^2)}$$

$$= \pi^{\omega} \, \varepsilon^{2(\omega-2)} \quad \frac{4-2\,\omega}{2\,\omega-2} \Gamma(2-\omega)$$

$$\xrightarrow[\substack{\varepsilon \to 0}]{\lim \; \omega \to 2} \pi^2 . \qquad \text{(G-11)}$$

Obviously non-zero contribution (G-11) has come from the ultraviolet region of integration.

Next, consider the following integral

$$J \equiv \int \frac{d^4k}{k^4} F((p-k)^2) \left[\frac{2p^2+k^2}{2p.k-k^2} + \frac{n^2(2p.k-k^2)}{2(k.n)^2} \right]. \qquad \text{(G-12)}$$

Again with the usual assumption that F is a well defined function and employing the regulator prescriptions mentioned

earlier, after shifting the variable and performing the angular integration (G-12) takes the form

$$J = 2\pi^2 \int_0^\infty dk^2\ F(k^2)\Big[\frac{1}{2\varepsilon^2(p^2-k^2)}\big[\{(k^2+p^2+\varepsilon^2)^2-4k^2p^2\}^{1/2}-|k^2-p^2|$$

$$+\frac{1}{p^2-k^2}\left(\frac{\varepsilon^2}{4p^2}-\frac{1}{2}\right)-\frac{1}{4p^2(p^2-k^2)}\{(p^2+k^2+\varepsilon^2)^2-4k^2p^2\}^{1/2}$$

$$+\frac{k^2+p^2}{4p^2(p^2-k^2)}-\frac{p^2-k^2}{2\varepsilon^2}\Big[\frac{1}{|k^2-p^2|}-\frac{1}{\{(k^2+p^2+\varepsilon^2)^2-4k^2p^2\}^{1/2}}\Big]\Big]. \tag{G-13}$$

From our experience of dealing with the integral (G-4), we expect that the first derivative of δ-function, in addition to δ-function itself, will occur when the limit $\varepsilon \to 0$ is taken inside the integral. No higher derivative of δ-function should occur. This can be seen by taking an explicit form of F, like $F((p-k)^2) = ((p-k)^2-p^2)^n/((p-k)^4+p^4)^n$, $n=2,3,\ldots.,$ in which case no contribution should come from δ-function or any of its derivatives. Thus if the limit $\varepsilon \to 0$ is taken inside the integral (G-13), then it will take the form

$$J = \lim_{\kappa\to 0} 2\pi^2\Big[-\int_0^{p^2-\kappa^2} F(k^2)\,\frac{dk^2}{2p^2}+\int_{p^2-\kappa^2}^{p^2+\kappa^2} dk^2F(k^2)$$

$$\{a\,\delta'(k^2-p^2)+b\,\delta(k^2-p^2)\}\Big] \tag{G-14}$$

where the derivative of δ-function is with respect to k^2.

Explicit calculation with a couple of simple algebraic functions for $F(k^2)$ in Equations (G-13) and (G-14) shows that $a = 2p^2$ and $b = -1$. Thus, finally (G-12) reduces to

$$J = -\frac{\pi^2}{p^2} \int_0^{p^2} dk^2\ F(k^2) - 2\pi^2\ F(p^2) - 4\pi^2 p^2\ \frac{\partial F(p^2)}{\partial p^2}. \qquad \text{(G-15)}$$

Here again we can not apply Equation (G-14) for F = constant. However integrands of (G-12) and (G-4) have the same ultraviolet behavior except for the difference of overall sign. Hence their ultraviolet contributions will cancel each other in Equations (G-2b) and (G-3b) for $F = F(p^2)$ and $G(p^2)$ respectively, and hence Equations (G-8) and (G-14) are sufficient there for F = constant, as well. Using Equations (G-8) and (G-14) in Equations (G-2b) and (G-3b), we get

$$1 = p^2F(p^2) + \beta M^2 p^2\ \frac{\partial F(p^2)}{\partial p^2} + Y_F(p^2) \qquad \text{(G-16)}$$

and

$$0 = p^2G(p^2) + \frac{\beta}{2}\ \frac{M^2}{p^2} \int_0^{p^2} dk^2 G(k^2) - \frac{\beta M^2}{2}\ G(p^2)$$

$$+\ \beta M^2 p^2\ \frac{\partial G(p^2)}{\partial p^2} + Y_G(p^2)\ , \qquad \text{(G-17)}$$

where $\beta = AC_2 Z(M)\ \frac{g_o^2}{4\pi^2}$. Analogous equations for the renormalized quark propagator can be written with the alteration $F \to F_R$, $G \to G_R$, $Y_F \to Y_{FR}$ $(1-Y_F = Z_2(1-Y_{FR}))$ and $Y_G \to Y_{GR}$. Next, we shall drop Y_{FR} and Y_{GR}, and denote renormalized functions without the subscript R. Then the equation for F

can be converted into the following second order differential equation ($y = -p^2/\beta M^2$):

$$y F'' + (1-y) F' - F = 0 . \qquad \text{(G-18)}$$

Independent solutions of Equation (G-18) are the confluent hypergeometric functions $\Phi(1;1; -p^2/\beta M^2)$ and $\Psi(1;1;-p^2/\beta M^2)$. When substituted back in Equation (G-16) (with $Y_F = 0$), first solution is ruled out while for the second one, the multiplicative constant is fixed:

$$F(p^2) = -\frac{1}{\beta M^2} \Psi(1;1; -p^2/\beta M^2). \qquad \text{(G-19)}$$

Similarly, the equation for G can be converted into the following second order differential equation:

$$y G'' - (y - 3/2) G' - 2G = 0 . \qquad \text{(G-20)}$$

Independent solutions in this case are $\Phi(2;3/2;-p^2/\beta M^2)$ and $(-p^2/\beta M^2)^{-1/2} \Phi(3/2;1/2; -p^2/\beta M^2)$. When substituted in Equation (3-14), second solution gives,

$$\text{effective mass} = S^{-1}(0) = G^{-1}(0) = 0,$$

and hence is ruled out. Thus we get

$$G(p^2) = C_1 \Phi(2;3/2; -p^2/\beta M^2). \qquad \text{(G-21)}$$

Appendix H

SOME ANGULAR INTEGRATIONS USED IN CALCULATION OF PAULI FORM FACTOR

We shall perform the following integration:

$$\int \frac{d^4k}{(k-p)^4} \frac{G(k'^2)-G(k^2)}{k'^2-k^2}\gamma_\lambda \, (\gamma_\mu \not{q}-2k_\mu)\gamma_\sigma[\delta_{\lambda\sigma} - \frac{(k-p)_\lambda n_\sigma+(k-p)_\sigma n_\lambda}{n.(k-p)}$$

$$+ \frac{(k-p)_\lambda (k-p)_\sigma n^2}{(n.(k-p))^2}], \; k' = k+q. \qquad \text{(H-1)}$$

First consider

$$I_{\lambda\sigma} = \int \frac{d\,\Omega_k}{(k-p)^4(q^2+2k.q)} [\delta_{\lambda\sigma} - \frac{(k-p)_\lambda n_\sigma + (k-p)_\sigma n_\lambda}{n.(k-p)}$$

$$+ \frac{(k-p)_\lambda (k-p)_\sigma n^2}{(n.(k-p))^2}]. \qquad \text{(H-2)}$$

Choose the gauge $n.p = n.q = 0$ and assume that $p_\mu = \kappa\, q_\mu$, then

$$I_1 = \int \frac{d\,\Omega_k}{(k-p)^4 \, (q^2+2k.q)}$$

$$= \frac{\kappa}{k^2+p^2+pq} [\frac{2\pi^2}{k^2_> |k^2-p^2|} + \frac{1}{k^2+p^2+pq} \{ \frac{2\pi^2}{k^2_>} + \frac{S(k,q)}{\kappa} \}] \qquad \text{(H-3)}$$

where we have used ($k^2_> = \max\,(k^2,p^2)$)

$$\int \frac{d\,\Omega_k}{(k-p)^2} = \frac{2\pi^2}{k_>^2}\,, \quad \int \frac{d\,\Omega_k}{(k-p)^4} = \frac{2\pi^2}{k_>^2\,|k^2-p^2|}\,,$$

$$\int \frac{d\,\Omega_k}{q^2+2k.q} \to P\int \frac{d\,\Omega_k}{q^2+2k.q} = \frac{\pi^2}{k^2}[1-(1-\frac{4k^2}{q^2})^{1/2}\,\theta(\frac{q^2}{4}-k^2)\,]$$

$$\equiv S(k,q).$$

In the last integral (which is not defined for $2k > q$) we have taken the principal value because in the context of the whole integral (H-1), it is the principal value which is relevant.

For those integrals in which (n.k) or its power occurs in denominator, we shall use the standard principal value prescription[64], which in simple cases reads

$$(k.n)^{-1} \to \frac{1}{2}\{[(k.n)+i\varepsilon']^{-1}+[(k.n)-i\varepsilon']^{-1}\}$$

$$\Rightarrow \int_0^\pi \frac{\text{Sin}\theta\,d\theta}{\text{Cos}\theta} = 0 \qquad \text{(H-4a)}$$

$$(k.n)^{-2} \to \frac{1}{2}\{[(k.n)+i\varepsilon']^{-2}+[(k.n)-i\varepsilon']^{-2}\}$$

$$\Rightarrow \int_0^\pi \frac{\text{Sin}\theta\,d\theta}{\text{Cos}^2\theta} = -2. \qquad \text{(H-4b)}$$

Thus,

$$\int \frac{d\,\Omega_k}{(k-p)^4(q^2+2k.q)(n.k)} = 0\,, \int \frac{d\Omega_k\,k_\lambda}{(k-p)^4(q^2+2k.q)(n.k)} = I_1\,\frac{n_\lambda}{n^2}$$

(H-5)

and

$$I_2 = \int \frac{d\Omega_k\ n^2}{(k-p)^4(q^2+2k.q)(n.k)^2}$$

$$= \frac{-4\pi^2\kappa}{k^2+p^2+pq}\ \left[\frac{k^2+p^2}{k^2|(k^2-p^2)^3|} + \frac{1}{k^2+p^2+pq}\left\{\frac{1}{k^2|k^2-p^2|}\right.\right.$$

$$\left.\left.+\frac{\theta(q^2/4-k^2)}{\kappa k^2(q^4-4k^2q^2)^{1/2}}\right\}\right], \qquad \text{(H-6)}$$

where we have used

$$\int \frac{d\Omega_k\ n^2}{(k-p)^4(n.k)^2} = -\frac{4\pi^2(k^2+p^2)}{k^2|(k^2-p^2)^3|},$$

$$\int \frac{d\Omega_k\ n^2}{(k-p)^2(n.k)^2} = -\frac{4\pi^2}{k^2|k^2-p^2|},$$

$$\int \frac{d\Omega_k\ n^2}{(q^2+2k.q)(n.k)^2} \rightarrow P\int \frac{d\Omega_k\ n^2}{(q^2+2k.q)(n.k)^2} = -\frac{4\pi^2\theta(q^2/4-k^2)}{k^2(q^4-4k^2q^2)^{1/2}}.$$

In the last integral we have again taken the principal value for the same reason as explained above.

$$\int \frac{d\Omega_k\ k_\lambda}{(k-p)^4(q^2+2k.q)(n.k)^2} = -\frac{2\pi^2(k^2+p^2)q_\lambda}{n^2q^2k^2|(k^2-p^2)^3|} - \frac{I_2q_\lambda}{2n^2} \qquad \text{(H-7)}$$

$$\int \frac{d\Omega_k\ k_\lambda\ k_\sigma}{(k-p)^4(q^2+2k.q)(n.k)^2} = A\ q_\lambda q_\sigma + B\ n_\lambda n_\sigma + C\ \delta_{\lambda\sigma} \qquad \text{(H-8)}$$

$$A = \frac{1}{2q^2}\left[\frac{I_1}{n^2} - \frac{k^2}{n^2} I_2 + 3\frac{I_3}{p^2}\right],$$

$$B = \frac{1}{2n^2}\left[3\frac{I_1}{n^2} - \frac{k^2}{n^2} I_2 + \frac{I_3}{p^2}\right],$$

$$C = -\frac{1}{2}\left[\frac{I_1}{n^2} - \frac{k^2}{n^2} I_2 + \frac{I_3}{p^2}\right]; \qquad \text{(H-8a)}$$

$$I_3 = \frac{1}{4}\int \frac{d\,\Omega_k}{(q^2+2k.q)(n.k)^2}\left[1 - \frac{2(k^2+p^2)}{(k-p)^2} + \frac{(k^2+p^2)^2}{(k-p)^4}\right]$$

$$= -\frac{\pi^2\theta(q^2/4-k^2)}{n^2k^2(q^4-4k^2q^2)^{1/2}} + \frac{p^2+k^2}{2n^2}\,\frac{4\pi^2\kappa}{k^2+p^2+pq}\left\{\frac{1}{k^2|k^2-p^2|}\right.$$

$$\left. + \frac{\theta(q^2/4-k^2)}{\kappa k^2(q^4-4k^2q^2)^{1/2}}\right\} + \frac{(k^2+p^2)^2}{4n^2} I_2.$$

(H-8b)

. Equations (H-3), (H-5)-(H-8b) are substituted in .tion (H-2), we get

$$I_{\lambda\sigma} = \frac{q_\lambda q_\sigma}{q^2}\,\frac{1}{k^2}\left[-\frac{\pi^2\kappa(8p^2+5pq)}{2p^2(k^2+p^2+pq)^2}\theta(p^2-k^2)\right.$$

$$+ \frac{\pi^2\kappa(8p^2+3pq)}{2p^2(k^2+p^2+pq)^2}\theta(k^2-p^2)$$

$$+ \frac{\pi^2}{2} \quad \frac{1}{(k^2+p^2+pq)^2} \quad -2\pi^2 \{ \frac{(q^4-4k^2q^2)^{1/2}}{q^2(k^2+p^2+pq)^2}$$

$$+ \frac{2}{(q^4-4k^2q^2)^{1/2}(k^2+p^2+pq)} \} \theta(q^2/4-k^2)]. \tag{H-2'}$$

Next, we consider

$$I_\mu = \int \frac{d\,\Omega_k\, k_\mu}{(k-p)^4(q^2+2k.q)} [2 + \frac{n^2(k-p)^2}{(n.k)^2}]$$

$$= A_1 q_\mu/q^2 + B_1\, n_\mu/n^2 \tag{H-9}$$

where

$$A_1 = \frac{2\pi^2}{k^2} [- \frac{\theta(p^2-k^2)}{p^2} + \frac{q^2}{4\pi^2} \frac{\pi^2\kappa(8p^2+4pq)}{2p^2(k^2+p^2+pq)^2} \theta(p^2-k^2)$$

$$- \frac{q^2}{4\pi^2} \frac{\pi^2\kappa(8p^2+4pq)}{2p^2(k^2+p^2+pq)^2} \theta(k^2-p^2) + \frac{q^2}{2} \{ \frac{(q^4-4k^2q^2)^{1/2}}{q^2(k^2+p^2+pq)^2}$$

$$+ \frac{2}{(q^4-4k^2q^2)^{1/2}(k^2+p^2+pq)} \} \theta(q^2/4-k^2)]. \tag{H-9a}$$

B_1 is not of our interest. From Equations (H-2), (H-2'), (H-9) and (H-9a), we get

$$\int \frac{d^4k}{(k-p)^4} \frac{G(k^2)}{k'^2-k^2} \gamma_\lambda (\gamma_\mu \not{q} - 2k_\mu) \gamma_\sigma [\delta_{\lambda\sigma} - \frac{(k-p)_\lambda n_\sigma + (k-p)_\sigma n_\lambda}{n.(k-p)} + \frac{(k-p)_\lambda (k-p)_\sigma n^2}{(n.(k-p))^2}]$$

$$= -\frac{\not{q}\gamma_\mu}{2} [- \int_0^{p^2} dk^2 \frac{\pi^2 \kappa (8p^2+4pq)\, G(k^2)}{2p^2(k^2+p^2+pq)^2} + \int_{p^2}^{\infty} dk^2 \frac{\pi^2 \kappa (8p^2+4pq) G(k^2)}{2p^2(k^2+p^2+pq)^2} - 2\pi^2 \int_0^{q^2/4} dk^2 \{ \frac{(q^4-4k^2q^2)^{1/2} G(k^2)}{q^2(k^2+p^2+pq)^2}$$

$$+ \frac{2G(k^2)}{(q^4-4k^2q^2)^{1/2}(k^2+p^2+pq)} \}] - 2\pi^2 \frac{q_\mu}{q^2} [\int_0^{p^2} dk^2\, G(k^2) \{ -\frac{1}{p^2} + \frac{q^2}{4\pi^2} \frac{\pi^2 \kappa (8p^2+4pq)}{2p^2(k^2+p^2+pq)^2} \} - \int_{p^2}^{\infty} dk^2 \frac{q^2 \pi^2 \kappa}{4\pi^2}$$

$$\times \frac{(8p^2+4pq)\, G(k^2)}{2p^2(k^2+p^2+pq)^2} + \int_0^{q^2/4} dk^2 \{ \frac{(q^4-4k^2q^2)^{1/2} G(k^2)}{q^2(k^2+p^2+pq)^2}$$

$$+ \frac{2G(k^2)}{(q^4-4k^2q^2)^{1/2}(k^2+p^2+pq)} \} \frac{q^2}{2}] + n_\mu \text{ terms.} \tag{H-10}$$

Next, consider the following integration

$$\int \frac{d^4k}{(k-p)^4} \frac{G(k'^2)}{k'^2-k^2} \gamma_\lambda (\gamma_\mu \not{q} - 2k_\mu) \gamma_\sigma [\delta_{\lambda\sigma} - \frac{(k-p)_\lambda n_\sigma + (k-p)_\sigma n_\lambda}{n.(k-p)} + \frac{(k-p)_\lambda (k-p)_\sigma n^2}{(n.(k-p))^2}]$$

$$= \int \frac{d^4k}{(k-p')^4} \frac{G(k^2)}{2k.q-q^2} \gamma_\lambda (\gamma_\mu \not{q} - 2k_\mu + 2q_\mu) \gamma_\sigma [\delta_{\lambda\sigma} - \frac{(k-p')_\lambda n_\sigma + (k-p')_\sigma n}{n.k}$$

$$+ \frac{(k-p')_\lambda (k-p')_\sigma n^2}{(n.k)^2}] + \delta\text{-function part (if any)}, \qquad \text{(H-11)}$$

where $p' = p+q$. Also define $p'_\mu = \kappa' q_\mu$. Then,

$$I'_{\lambda\sigma} = - \int \frac{d\Omega_k}{(k-p')^4 (q^2+2k.(-q))} [\delta_{\lambda\sigma} - \frac{(k-p')_\lambda n_\sigma + (k-p')_\sigma n_\lambda}{n.k}$$

$$+ \frac{(k-p')_\lambda (k-p')_\sigma n^2}{(n.k)^2}]$$

$$= - I_{\lambda\sigma}\{p \to p', \ qp \to -qp', \ \kappa \to -\kappa'\}$$

$$= - \frac{q_\lambda q_\sigma}{q^2} \frac{\pi^2}{k^2} [\frac{\kappa'(8p'^2-5p'q)}{2p'^2(k^2+p'^2-p'q)^2} \theta(p'^2-k^2)$$

$$- \frac{\kappa'(8p'^2-3p'q)}{2p'^2(k^2+p'^2-p'q)^2} \theta(k^2-p'^2) + \frac{1}{2} \frac{1}{(k^2+p'^2-p'q)^2}$$

$$- 2 \{ \frac{(q^4-4k^2q^2)^{1/2}}{q^2(k^2+p'^2-p'q)^2} + \frac{2}{(q^4-4k^2q^2)^{1/2}(k^2+p'^2-p'q)} \} \theta(q^2/4-k^2)]$$

(H-12)

Here, $k^2+p'^2-p'q = k^2+p^2+pq$. It can be easily checked that

$$\int I_{\lambda\sigma} k^2 \, dk^2 = \int I'_{\lambda\sigma} k^2 \, dk^2 = 0 .$$

Hence by shifting the variable, as in Equation (H-11), the apparent (infrared) divergent nature of integral does not

introduce any δ-function.

In a similar way,

$$I'_\mu = \int \frac{d\Omega_k \, k_\mu}{(k-p')^4 (2k.q-q^2)} \left[2 + \frac{(k-p')^2 n^2}{(n.k)^2}\right] = A'_1 \frac{q_\mu}{q^2} + B'_1 \frac{n_\mu}{n^2}$$

(H-13)

where

$$A'_1 = A_1(p \to p', \; qp \to -qp', \; \kappa \to -\kappa')$$

$$= \frac{2\pi^2}{k^2} \Big[-\frac{\theta(p'^2-k^2)}{p'^2} - \frac{q^2}{4\pi^2} \frac{\pi^2 \kappa'(8p'^2-4p'q)}{2p'^2(k^2+p'^2-p'q)^2} \theta(p'^2-k^2)$$

$$+ \frac{q^2}{4\pi^2} \frac{\pi^2 \kappa'(8p'^2-4p'q)}{2p'^2(k^2+p'^2-p'q)^2} \theta(k^2-p'^2) + \frac{q^2}{2}$$

$$\times \left\{ \frac{(q^4-4k^2q^2)^{1/2}}{q^2(k^2+p'^2-p'q)^2} + \frac{2}{(q^4-4k^2q^2)^{1/2}(k^2+p'^2-p'q)} \right\}$$

$$\times \, \theta(q^2/4-k^2) \Big].$$

(H-13a)

Substituting Equations (H-12)-(H-13a) in Equation (H-11), we get

$$\int \frac{d^4k}{(k-p)^4} \frac{G(k'^2)}{k'^2-k^2} \gamma_\lambda (\gamma_\mu \not{k} - 2k_\mu) \gamma_\sigma \Big[\delta_{\lambda\sigma} - \frac{(k-p)_\lambda n_\sigma + (k-p)_\sigma n_\lambda}{n.(k-p)}$$

$$+ \frac{(k-p)_\lambda (k-p)_\sigma n^2}{(n.(k-p))^2} \Big]$$

$$= \frac{\not{q}\gamma_\mu + 2q_\mu}{2} \left[\int_0^{p'^2} dk^2 G(k^2) \frac{\pi^2 \kappa'(8p'^2 - 4p'q)}{2p'^2(k^2+p^2+pq)^2} - \int_{p'^2}^{\infty} dk^2 G(k^2) \right.$$

$$\times \frac{\pi^2 \kappa'(8p'^2-4p'q)}{2p'^2(k^2+p^2+pq)^2} - 2\pi^2 \int_0^{q^2/4} dk^2\, G(k^2) \left\{ \frac{(q^4-4k^2q^2)^{1/2}}{q^2(k^2+p^2+pq)^2} \right.$$

$$\left.\left. + \frac{2}{(q^4-4k^2q^2)^{1/2}(k^2+p^2+pq)} \right\} \right] - \frac{q_\mu}{2} \left[\int_0^{p'^2} dk^2\, G(k^2) \right.$$

$$\times \frac{\pi^2 \kappa'(8p'^2-4p'q)}{2p'^2(k^2+p^2+pq)^2} - \int_{p'^2}^{\infty} dk^2 G(k^2) \frac{\pi^2 \kappa'(8p'^2-4p'q)}{2p'^2(k^2+p^2+pq)^2}$$

$$- 2\pi^2 \int_0^{q^2/4} dk^2 \left\{ G(k^2) \frac{(q^4-4k^2q^2)^{1/2}}{q^2(k^2+p^2+pq)^2} + \frac{2G(k^2)}{(q^4-4k^2q^2)^{1/2}(k^2+p^2+pq)} \right\}$$

$$- \frac{q_\mu}{q^2} \frac{2\pi^2}{p'^2} \int_0^{p'^2} dk^2\, G(k^2) + n_\mu \text{ terms.} \qquad \text{(H-14)}$$

Substituting Equations (H-10) and (H-14) in Equation (H-1), we get

$$\int \frac{d^4k}{(k-p)^4} \frac{G(k'^2)-G(k^2)}{k'^2 - k^2} \gamma_\lambda(\gamma_\mu \not{q} - 2k_\mu)\gamma_\sigma \left[\delta_{\lambda\sigma} - \frac{(k-p)_\lambda n_\sigma + (k-p)_\sigma n_\lambda}{n.(k-p)} + \frac{(k-p)_\lambda (k-p)_\sigma n^2}{(n.(k-p))^2} \right]$$

$$= q_\nu [\gamma_\nu, \gamma_\mu] \left\{ \pi^2 \kappa' \int_{p^2}^{p'^2} dk^2 \frac{G(k^2)(2p'^2 - p'q)}{p'^2(k^2+p^2+pq)^2} - \pi^2 \int_0^{q^2/4} dk^2 G(k^2) \times \right.$$

$$\left. \left[\frac{(q^4-4k^2q^2)^{1/2}}{q^2(k^2+p^2+pq)^2} + \frac{2}{(q^4-4k^2q^2)^{1/2}(k^2+p^2+pq)} \right] \right\}$$

$$- \frac{q_\mu}{q^2} 2\pi^2 \left[\frac{1}{p'^2} \int_0^{p'^2} dk^2 G(k^2) - \frac{1}{p^2} \int_0^{p^2} dk^2 \, G(k^2) \right] + n_\mu \text{terms.}$$

(H-15)

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